10.1.4 How can I solve it?

Fraction Busters

In Lesson 10.1.3, you learned a powerful new method to help solve complicated equations: rewriting the equation first to create a simpler, equivalent equation. Today you will continue to solve new, complicated equations and will focus specifically on equations with fractions. As you solve these new problems, look for ways to **connect** today's work with what you have learned previously.

10-34. Examine the equation below.

$$\frac{5x}{3} + \frac{15}{2} = \frac{5}{2}$$

a. Solve the equation by first finding an equivalent equation without fractions. Check your solution(s).

b. Often, this method of eliminating fractions from an equation is called the **Fraction Busters Method** because the multiplication of the equation by a common denominator or several of the denominators eliminates ("busts") the fractions. The result is an equation with no fractions.

By what number (or numbers) did you multiply both sides of the equation in part (a) to eliminate the fractions? How did you choose that number? Is it the smallest number that would eliminate all of the fractions?

10-35. Work with your team to solve each of the equations below by first finding an equivalent equation that contains no fractions. Each problem presents new challenges and situations. Be ready to **justify** how you solved each problem and share why you did what you did with the class. Remember to check each solution.

a. $\frac{x}{4} - \frac{x}{6} = \frac{2}{3}$

b. $\frac{5}{x} - 2x = 3$

c. $\frac{-2x+1}{3} - \frac{x+3}{7} = 8$

d. $\frac{x+3}{x-2} + 2 = \frac{x+5}{x-2}$

10-36. Now examine the equation below.

$$\frac{4+p}{p^2+2p-8} + 3 = \frac{4}{p-2}$$

a. What values of p are not allowed? Show how you know.

b. Use your new skills to rewrite the equation above so that it has no fractions. Then solve the new equation. Check your solution(s). What happened?

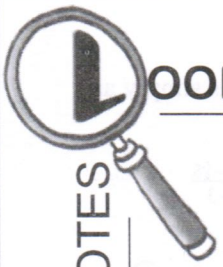
10-37. Solve the equations below by first changing each equation to a simpler, equivalent equation. Check your solution(s).

a. $50x^2 + 200x = -150$

b. $\frac{a}{9} + \frac{1}{a} = \frac{2}{3}$

c. $1.2m - 0.2 = 3.8 + m$

d. $\frac{2}{x+5} + \frac{3x}{x^2+2x-15} = \frac{4}{x-3}$



LOOKING DEEPER

MATH NOTES

Solving Equations with Algebraic Fractions (also known as Fraction Busters)

Example: Solve $\frac{x}{3} + \frac{x}{5} = 2$ for x.

This equation would be much easier to solve if it had no fractions. Therefore, the first goal is to find an equivalent equation that has no fractions.

$$\frac{x}{3} + \frac{x}{5} = 2$$

The lowest common denominator of $\frac{x}{3}$ *and* $\frac{x}{5}$ *is* 15.

To eliminate the denominators, multiply both sides of the equation by the common denominator. In this example, the lowest common denominator is 15, so multiplying both sides of the equation by 15 eliminates the fractions. Another approach is to multiply both sides of the equation by one denominator and then by the other.

$$15 \cdot \left(\frac{x}{3} + \frac{x}{5}\right) = 15 \cdot 2$$

$$15 \cdot \frac{x}{3} + 15 \cdot \frac{x}{5} = 15 \cdot 2$$

Either way, the result is an equivalent equation without fractions:

$$5x + 3x = 30$$
$$8x = 30$$

The number used to eliminate the denominators is called a **fraction buster**. Now the equation looks like many you have seen before, and it can be solved in the usual way.

$$x = \frac{30}{8} = \frac{15}{4} = 3.75$$

$$\frac{3.75}{3} + \frac{3.75}{5} = 2$$

Once you have found the solution, remember to check your answer.

$$1.25 + 0.75 = 2$$

Review & Preview

10-38. Solve the equations below by first changing each equation to a simpler equivalent equation. Check your solutions.

a. $3000x - 2000 = 10{,}000$

b. $\frac{x^2}{2} + \frac{3x}{2} - 5 = 0$

c. $\frac{5}{2}x - \frac{1}{3} = 13$

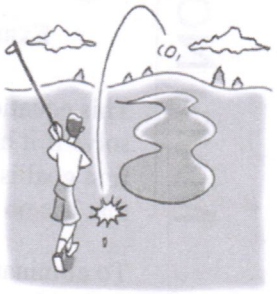
d. $\frac{3}{10} + \frac{2x}{5} = \frac{1}{2}$

10-39. Multiply or divide the expressions below. Express your answers as simply as possible.

a. $\frac{5x^2-11x+2}{x^2+8x+16} \cdot \frac{x^2+10x+24}{10x^2+13x-3}$

b. $\frac{6x+3}{2x-3} \div \frac{3x^2-12x-15}{2x^2-x-3}$

10-40. To avoid a sand trap, a golfer hits a ball so that its height is represented by the equation $h = -16t^2 + 80t$, where h is the height measured in feet and t is the time measured in seconds.



a. When does the ball land on the ground?

b. What is the maximum height of the ball during its flight?

10-41. Write and solve an equation (or a system of equations) for the following situation. Be sure to define your variables.

Each morning, Jerry delivers two different newspapers: the *Times* and the *Star*. The *Times* weighs $\frac{1}{2}$ a pound and the *Star* weighs $\frac{1}{4}$ a pound. If he delivers a total of 27 newspapers that weigh a total of $11\frac{1}{2}$ pounds, how many *Times* newspapers does he deliver?

10-42. Graph the system of inequalities below on graph paper. Shade the region that represents the solution.

$$y \geq x^2 - 4$$
$$y \leq -x^2 + 4$$

10-43. **Multiple Choice:** $x = 2$ is a solution to which of the equations or inequalities below?

a. $\frac{x-4}{3} = \frac{x}{15}$ b. $(x-2)^2 < 0$ c. $|3x-8| \geq -1$ d. $\sqrt{x+2} = 16$