SIMPLE HARMONIC MOTION

DEFINITION

This is the periodic motion of a body or particle along a straight line such that the acceleration of the body is directed towards a fixed point .

A particle undergoing simple harmonic motion will move to and fro in a straight line under the influence of a force. This influential force is called a restoring force as it always directs the particle back to its equilibrium position.

Examples of simple harmonic motions are

i. loaded test tube in a liquid

ii Mass on a string

iii The simple pendulum

As the particle P moves round the circle once, it sweeps through an angle θ = 360 (or 2π radians) in the time T the period of motion. The rate of change of the angle θ with time (t) is known as the angular velocity ω

Angular velocity (ω) is defined by

ω = angle turned through by the body

Time taken

$\omega = \theta / t$ (rad /sec)

$\theta = \omega t$

This is similar to the relation distance = uniform velocity x time (s= =vt) for motion in a straight line

As the angle is changing with time so is the arc length

S = z p

Changing with time. By definition θ in radians = s/r and hence

$S = r\theta$

A = r = radius of the circle

$s/t = r\theta /t = s/r /t$

$s/t = s/t \times 1/r = r \theta /t$

$v = r \omega$

The linear velocity v at any point ,Q whose distance from C the central point is x is given by

$V = \omega \sqrt{A2 – X2}$

The minimum velocity ,Vm corresponds to the point at X = 0 that is the velocity at the central point or centre of motion .

Hence, $V_m = \omega A$

Thus the maximum velocity of the SHM occurs at the centre of the motion (X=0) while the minimum velocity occurs at the extreme position of motion (x=A).

RELATIONSHIP BETWEEN LINEAR ACCELERATION AND ANGULAR VELOCITY

$X = A \cos \theta$

$\theta = \omega t$

$X = A \cos \omega t$

$dx = -\omega A \sin \omega t$

dt

$dv = -\omega 2 A \cos \omega t$

dt

$= -\omega 2X$

The negative sign indicates that the acceleration is always inwards towards C while the displacement is measured outwards from C.

Energy of simple harmonic motion

Forced vibration and resonance

ENERGY OF SIMPLE HARMONIC MOTION

Since force and displacement are involved, it follows that work and energy are involved in simple harmonic motion.

At any instant of the motion , the system may contain some energy as kinetic energy (KE) or potential energy(PE) .The total energy (KE + PE) for a body performing SHM is always conserved although it may change form between PE and KE .

When a mass is suspended from the end of a spring stretched vertically downwards and released, it oscillates in a simple harmonic motion .During this motion , the force tending to restore the spring to its elastic restoring force is simply the elastic restoring force which is given by

K is the force constant of the spring

The total work done in stretching the spring at distance y is given by

W = average force x displacement

$W = \frac{1}{2} ky \times y = \frac{1}{2} ky^2$

Thus the maximum energy total energy stored in the spring is given by

$W = \frac{1}{2} KA^2$

A = amplitude (maximum displacement from equilibrium position).

This maximum energy is conserved throughout the motion of the system.

At any stage of the oscillation, the total energy is

$W = \frac{1}{2} KA^2$

$W = \frac{1}{2} mv^2 + \frac{1}{2} ky^2$

$\frac{1}{2} mv^2 = \frac{1}{2} KA^2 - \frac{1}{2} ky^2$

$v^2 = k/m (A^2 - y^2)$

$V = \sqrt{k/m(A^2-y^2)}$

The constant K is obtained from

Hooke's law in which

$F = mg = ke$

Where e is the extension produced in the spring by a mass m

But $V = \omega\sqrt{A^2-X^2}$

Therefore $\omega = \sqrt{k/m}$

Hence the period, $T = 2\pi/\omega$

$T = 2\pi\sqrt{m/k}$

EXAMPLE

A body of mass 20g is suspended from the end of a spiral spring whose force constant is $0.4Nm^{-1}$. The body is set into a simple harmonic motion with amplitude 0.2m. Calculate:

The period of the motion

The frequency of the motion

The angular speed

The total energy

The maximum velocity of the motion

The maximum acceleration

SOLUTION

$T = 2\pi \sqrt{m/k} = 2\pi \sqrt{0.02/0.4} = 0.447\,\pi$ sec = 1.41 sec

$f=1/T = 1/1.41 = 0.71$Hz

$\omega = 2\pi f = 2\pi \times 0.71 = 4.46$ rad. S^{-1}

Total energy = $\frac{1}{2} KA^2 = \frac{1}{2} (0.4) (0.2)^2 = 0.008$ J

$\frac{1}{2} mv^2 = /12\ KA^2$

$V_m^2 = \dfrac{0.008 \times 2}{0.02}$

$= 0.8$

$V_m = 0.89$ m/s

Or $V = \omega A$

$= 4.462 \times 0.2$

$= 3.98 m/s^2$

FORCED VIBRATION AND RESONANCE

Vibrations resulting from the action of an external periodic force on an oscillating body are called forced vibrations. Every vibrating object possesses a natural frequency (fo) of vibration. This is the frequency with which the object will oscillate when it is left undisturbed after being set into vibration. The principle of the sounding board of a piano or the diaphragm of a loudspeaker is based on the phenomenon of forced vibrations.

Whenever the frequency of vibrating body acting on a system coincides with the natural frequency of the system, then the system is set into vibration with relatively large amplitude. This phenomenon is called resonance.

CLASSWORK

What is simple harmonic motion? Give four examples to illustrate simple harmonic motion

A particle moves round a circle of radius 10cm with a constant velocity of 20m/s, calculate the angular velocity

A particle undergoes simple harmonic motion with an amplitude of 5cm and an angular velocity of 10π rads-1, calculate: (i) the maximum velocity (ii) the velocity when it is 2cm from the equilibrium position (iii) the maximum acceleration of the particle (iv) the period of oscillation

Define the following terms: frequency, period, amplitude of simple harmonic motion. What is the relation between period and frequency?

ASSIGNMENT

SECTION A

The period of oscillation of a simple pendulum is 2.0s. Calculate the period if the length of the pendulum is doubled (a) 1.0s (b) 1.4s (c) 2.8s (d) 4.0s

The period of a body performing simple harmonic motion is 2.0s. If the amplitude of the motion is 3.5cm, calculate the maximum speed of the body (π=22/7) (a) 22.0cms-1 (b) 11.0cms-1 (c) 7.0cms-1 (d) 1.8cms-1

A pendulum bob, executing simple harmonic motion has 2cm and 12Hz ass amplitude and frequency respectively. Calculate the period of the motion (a) 2.00s (b) 0.83s (c) 0.08s (d) 0.06s

What is the angular speed of a body vibrating at 50 cycles per seconds (a) 200πrads-1 (b) 400 πrads-1 (c) 100 πrads-1 (d) 50 πrads-1

In the diagram below, the maximum potential energy of the swinging pendulum occurs at position(s) (a)1 and 5 (b) 2 and 4 (c) 3 0nly (d) 5 and 3

The motion of a body is simple harmonic if the: (a) acceleration is always directed towards a fixed point (b) path of motion is a straight line (c) acceleration is directed towards a fixed point and proportional to its distance from the point (d) acceleration is proportional to the square of the distance from a fixed point

Which of the following assumptions is made in a simple pendulum experiment? The (a) suspending string is inextensible (b) bob has a finite size (c) bob has a definite mass (d) initial angle of oscillation must be large

A simple pendulum has a period of 17.0s.When the length is shortened by 1.5m, its period is 8.5s. Calculate the original length of the pendulum (a) 1.5m (b) 2.0m (c) 3.0m (d) 4.0m

The period of oscillatory motion is defined as the (a) average of the time used in completing different numbers of oscillations (b) time to complete a number of oscillations (c) time to complete one oscillation (d) time taken to move from one extreme position to another

Which of the following correctly gives the relationship between linear speed V and angular speed ω of a body moving uniformly in a circle of radius r? (a) v= ωr (b) v= ω2r (c) v= ωr2 (d) v2= ωr

SECTION B

Derive the expression T=2πlg of the period of a simple pendulum. A simple pendulum has a period of 3.45 seconds. When the length of the pendulum is shortened by 1.0m, the period is 2.81 seconds. Calculate: (i) the original length of the pendulum (b) the value of the acceleration due to gravity.

A body of mass 0.5kg is attached to the end of a spring and the mass pulled down a distance 0.01m. Calculate: (i) the period of oscillation (ii) the maximum kinetic energy of mass (iii) kinetic and potential energy of the spring when the body is 0.04m below its centre of oscillation.(k=50Nm).

A body of mass 0.2kg is executing simple harmonic motion with amplitude of 20mm. The maximum force which acts upon it is 0.064N.Calculate (a) its maximum velocity (b) its period of oscillation.

(a) A body moving with simple harmonic motion in a straight line has a velocity v and acceleration, a, when the instantaneous displacement, x in cm, from its maximum position is given by: x=2.5sin 0.4πt. Determine the magnitude of maximum; (i) velocity (ii) acceleration

A mass m attached to a light spiral spring is caused to perform simple harmonic motion of frequency f=22πkm, where k is force constant of the spring (i) If m=0.30kg, k=30Nm-1 and the maximum displacement of the mass from the equilibrium position is 0.015m, calculate the maximum force acting on the system (ii) calculate the maximum kinetic energy of the system (iii) calculate the maximum tension in the spring during the motion [g=10ms-2, π=3.142]