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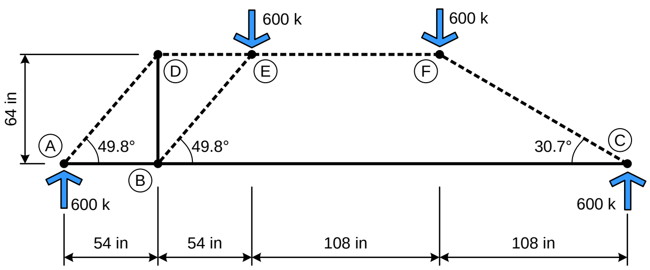
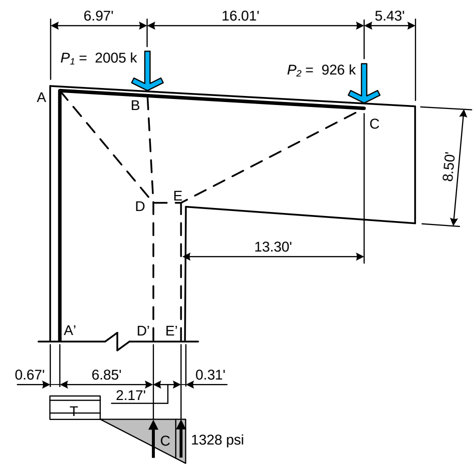
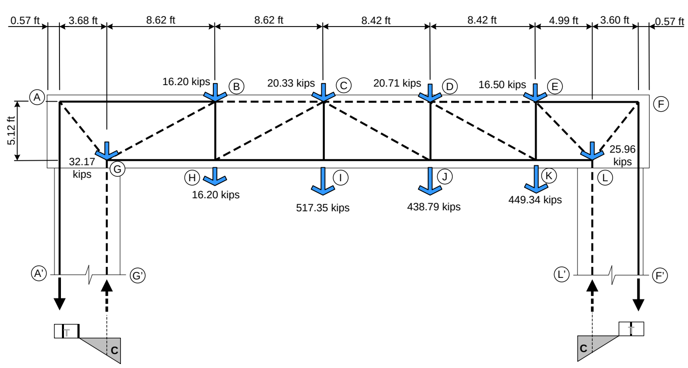
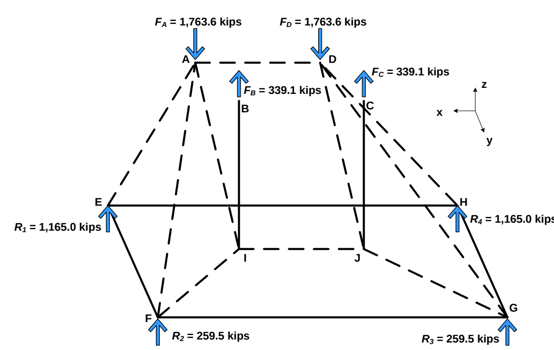
Strut-and-Tie Modeling (STM) for Concrete Structures









Design Examples

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FOREWORD

This Manual provides four design examples illustrating the application of the strut-and-tie method for a variety of structural configurations, including a simply-supported deep beam, a cantilever bent cap, an inverted-tee moment frame straddle bent cap, and a drilled shaft footing. Each design example is based on the 8th Edition of the *AASHTO LRFD Bridge Design Specifications*. This Manual is intended for state DOT bridge and structures engineers and practicing bridge engineers who are responsible for concrete bridge design and evaluation. This Manual will serve as a reference and a guide for engineers of all levels, including designers, consultants, reviewers, maintenance engineers, management engineers, and load rating engineers. This document is part of a training program that also includes a one-and-a-half-day instructor-led training (ILT) course.

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16. Abstract

This Manual provides four design examples illustrating the application of the strut-and-tie method for a variety of structural configurations, including a simply-supported deep beam, a cantilever bent cap, an inverted-tee moment frame straddle bent cap, and a drilled shaft footing. Each design example is based on the 8th Edition of the *AASHTO LRFD Bridge Design Specifications*. This Manual is intended for state DOT bridge and structures engineers and practicing bridge engineers who are responsible for concrete bridge design and evaluation. This Manual will serve as a reference and a guide for engineers of all levels, including designers, consultants, reviewers, maintenance engineers, management engineers, and load rating engineers. This document is part of a training program that also includes a one-and-a-half-day instructor-led training (ILT) course.

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SI* (MODERN METRIC) CONVERSION FACTORS

APPROXIMATE CONVERSIONS TO SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol
		LENGTH		
in	inches	25.4	millimeters	mm
ft	feet	0.305	meters	m
yd	yards	0.914	meters	m
mi	miles	1.61	kilometers	km
		AREA		
in^2	square inches	645.2	square millimeters	mm^2
ft^2	square feet	0.093	square meters	m^2
yd^2	square yard	0.836	square meters	m^2
ac	acres	0.405	hectares	ha
mi^2	square miles	2.59	square kilometers	km^2
		VOLUME		
fl oz	fluid ounces	29.57	milliliters	mL
gal	gallons	3.785	liters	L
ft^3	cubic feet	0.028	cubic meters	m^3
yd^3	cubic yards	0.765	cubic meters	m^3
		NOTE: volumes greater than 1000 L shall be shown in m^3		
		MASS		
oz	ounces	28.35	grams	g
lb	pounds	0.454	kilograms	kg
T	short tons (2000 lb)	0.907	megagrams (or "metric ton")	Mg (or "t")
		TEMPERATURE (exact degrees)		
^oF	Fahrenheit	5 (F-32)/9 or (F-32)/1.8	Celsius	^oC
		ILLUMINATION		
fc	foot-candles	10.76	lux	lx
fl	foot-Lamberts	3.426	candela/m^2	cd/m^2
		FORCE and PRESSURE or STRESS		
lbf	poundforce	4.45	newtons	N
lbf/in^2	poundforce per square inch	6.89	kilopascals	kPa

APPROXIMATE CONVERSIONS FROM SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol
		LENGTH		
mm	millimeters	0.039	inches	in
m	meters	3.28	feet	ft
m	meters	1.09	yards	yd
km	kilometers	0.621	miles	mi
		AREA		
mm^2	square millimeters	0.0016	square inches	in^2
m^2	square meters	10.764	square feet	ft^2
m^2	square meters	1.195	square yards	yd^2
ha	hectares	2.47	acres	ac
km^2	square kilometers	0.386	square miles	mi^2
		VOLUME		
mL	milliliters	0.034	fluid ounces	fl oz
L	liters	0.264	gallons	gal
m^3	cubic meters	35.314	cubic feet	ft^3
m^3	cubic meters	1.307	cubic yards	yd^3
		MASS		
g	grams	0.035	ounces	oz
kg	kilograms	2.202	pounds	lb
Mg (or "t")	megagrams (or "metric ton")	1.103	short tons (2000 lb)	T
		TEMPERATURE (exact degrees)		
^oC	Celsius	1.8C+32	Fahrenheit	^oF
		ILLUMINATION		
lx	lux	0.0929	foot-candles	fc
cd/m^2	candela/m^2	0.2919	foot-Lamberts	fl
		FORCE and PRESSURE or STRESS		
N	newtons	0.225	poundforce	lbf
kPa	kilopascals	0.145	poundforce per square inch	lbf/in^2

* SI is the symbol for the International System of Units. Appropriate rounding should be made to comply with Section 4 of ASTM E380. (Revised March 2003)

Visit http://www.fhwa.dot.gov/publications/convtabl.cfm for a 508 compliant version of this table.

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Acknowledgments

The examples contained in this document greatly benefited from the example problems previously developed in an implementation project sponsored by the Texas Department of Transportation (TxDOT). In this context, the contributions of Christopher Williams and Dean Deschenes have been invaluable and are gratefully acknowledged. Further, the support of Gregg Freeby, TxDOT's Bridge Division Director, is also gratefully acknowledged.

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Glossary

Available Length – The tie width for a CCT or CTT node over which the stirrups considered to carry the force in the tie can be spread.

Back Face – The face of a nodal zone at which neither a load, reaction, nor strut is located.

Beam or Bernoulli Region (B-Region) – Regions of concrete members in which Bernoulli's hypothesis of straight-line strain profiles, linear for bending and uniform for shear, applies.

Bearing Face – The face of a nodal zone at which a load or reaction is applied.

Bottle-shaped Strut – A strut that is wider at mid-length than at its ends.

CCC Node – A node where only struts intersect.

CCT Node – A node where a tie intersects the node in only one direction.

Concrete Efficiency Factor – A factor based on the node type (CCC, CCT, or CTT) and the node face (bearing face, back face, or strut-to-node interface) that is used to compute the limiting compressive stress at a node face.

Confinement Modification Factor – A factor based on the relative proportion of the supporting surface to the loaded area that is used to compute the limiting compressive stress at a node face.

Crack Control Reinforcement – Reinforcement, based on 0.003 times the effective area of the strut, intended to control the width of cracks and to ensure a minimum ductility for the member so that, if required, significant redistribution of internal stresses is possible.

CTT Node – A node where ties intersect in two different directions.

Curved Bar Node – A node resulting from bending a large reinforcing bar (such as No. 11, 14, or 18).

Development Length – The distance required to develop the specified strength of a reinforcing bar or prestressing strand.

Direct Strut Model – A strut-and-tie model in which a single direct strut is used to connect the nodes at two bearing faces.

Disturbed or Discontinuity Region (D-Region) – Regions of concrete members encompassing abrupt changes in geometry or concentrated forces in which strain profiles more complex than straight lines exist.

Interior Node – A node that is not located at the end reactions of the member.

Load and Resistance Factor Design – A reliability-based design methodology in which force effects caused by factored loads are not permitted to exceed the factored resistance of the components.

LRFD – Load and Resistance Factor Design.

Nodal Zone – The volume of concrete around a node that is assumed to transfer strut-and-tie forces through the node.

Node – A point in a strut-and-tie model where the axes of the struts, ties, and concentrated forces acting on the joint intersect.

Singular Node – A node with a clearly defined geometry.

Smeared Node – An interior node that is not bounded by a bearing plate.

STM – Strut-and-tie model; strut-and-tie modeling; strut-and-tie method.

Strut – A compression member in a strut-and-tie model representing the resultant of a parallel or a fan-shaped compression field.

Strut-and-Tie Method – A procedure used principally in regions of concentrated forces and geometric discontinuities to determine concrete proportions and reinforcement quantities and patterns based on an analytic model consisting of compression struts in the concrete, tensile ties in the reinforcement, and the geometry of nodes at their points of intersection.

Strut-and-Tie Model – A truss model of a member or of a D-Region in such a member, made up of struts and ties connected at nodes and capable of transferring the factored loads to the supports or to adjacent B-Regions.

Strut-to-Node Interface – The face of a nodal zone at which a strut is located.

Tie – A tension element in a strut-and-tie model.

Two Panel Model – A strut-and-tie model in which an intermediate vertical tie is introduced between the nodes at two bearing faces such that there are two panels between the nodes at the two bearing faces.

How to Use These Design Examples

This document provides four design examples illustrating the application of the strut-and-tie method for a variety of structural configurations, including the following:

- Design Example 1 – Simply-Supported Deep Beam
- Design Example 2 – Cantilever Bent Cap
- Design Example 3 – Inverted-Tee Moment Frame Straddle Bent Cap
- Design Example 4 – Drilled Shaft Footing

There are several characteristics that are common to all four design examples that are intended to benefit the designer as they use this document.

At the beginning of each design example is a table of contents specific to that example, as well as a flowchart of the various design steps. Each design step in the table of contents and in the flowchart is then clearly identified within the document.

Each design example contains a wealth of figures to illustrate and supplement the concepts being presented in the narrative. In addition, most of the design examples also contain several tables.

Each design example is based on Load and Resistance Factor Design (LRFD). As used in this document, *AASHTO LRFD* is used as an abbreviation of *AASHTO LRFD Bridge Design Specifications*. In addition, STM is used as an abbreviation for strut-and-tie model, strut-and-tie modeling, or strut-and-tie method. It is generally clear which is meant.

References to *AASHTO LRFD* articles, figures, tables, and equations are presented throughout the design examples. The designer can refer to those portions of *AASHTO LRFD* for clarification or for more information about the information being presented in the design examples.

In addition, tip-paragraphs are presented throughout the design examples in the following format:

> Tip-paragraphs are additional, supplemental information that may not necessarily be required to complete the design example but that is useful information for the designer to know as they seek to apply the strut-and-tie method to other structural configurations. Tip-paragraphs are set apart from all other paragraphs in two ways: (1) they are indented on the left and right, and (2) they are presented in narrow font, as illustrated in this paragraph.

Each design example is based on the 8th Edition of the *AASHTO LRFD Bridge Design Specifications*.

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STM Handout for Each Design Example

The following pages contain the basic strut-and-tie model layout for each of the four design examples.

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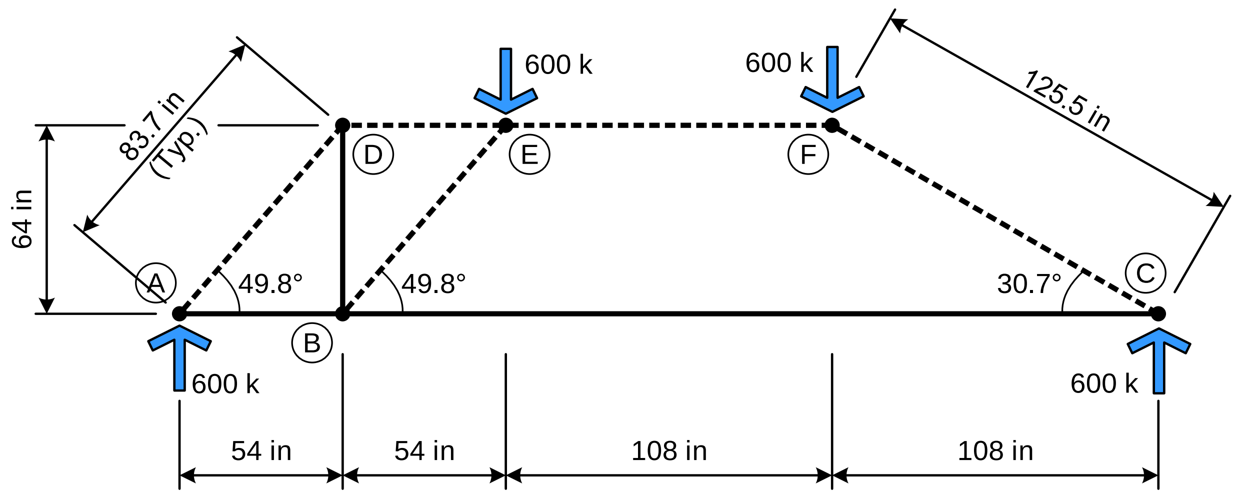
Design Example 1 – Simply-Supported Deep Beam:



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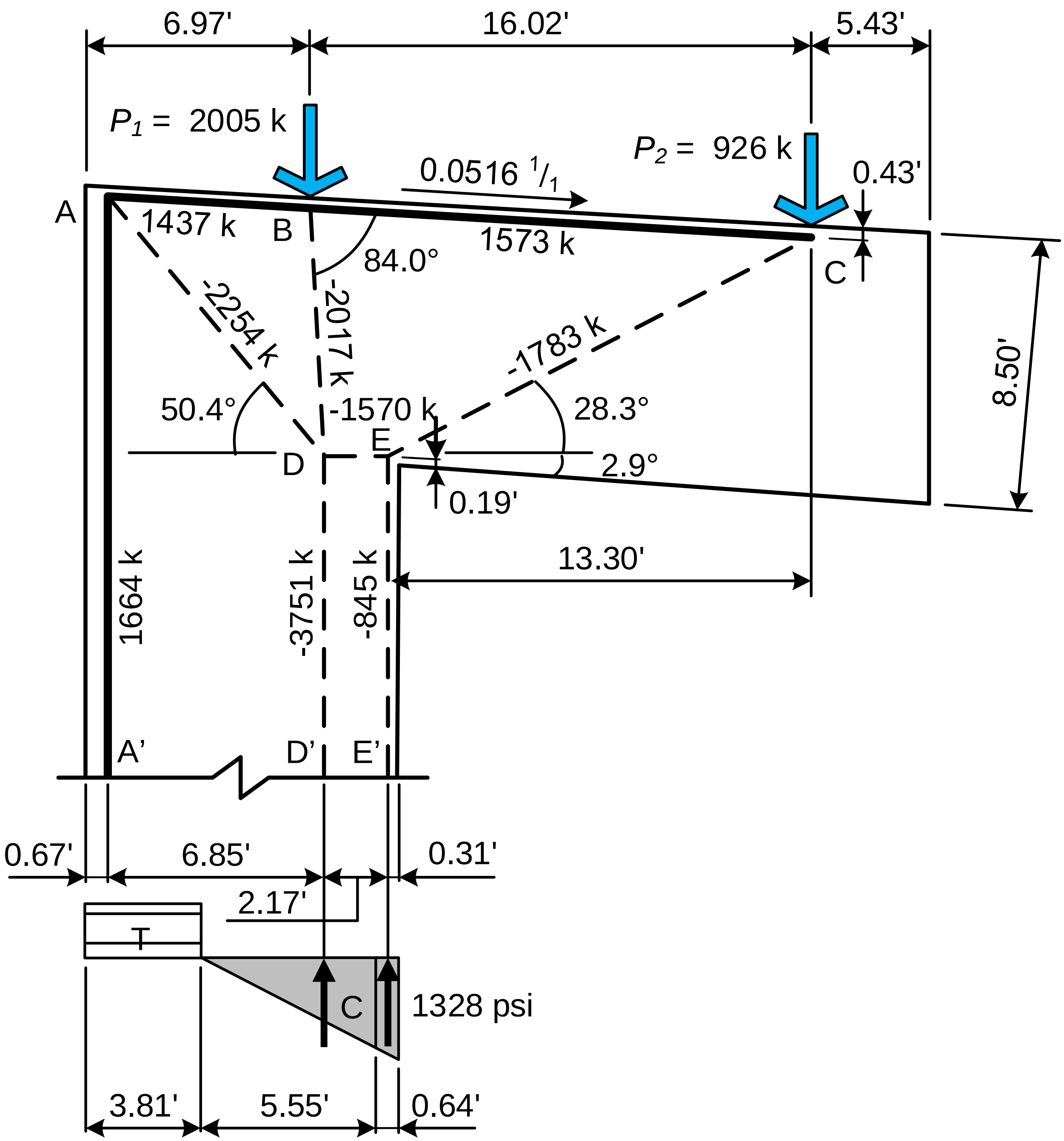
Design Example 2 – Cantilever Bent Cap:



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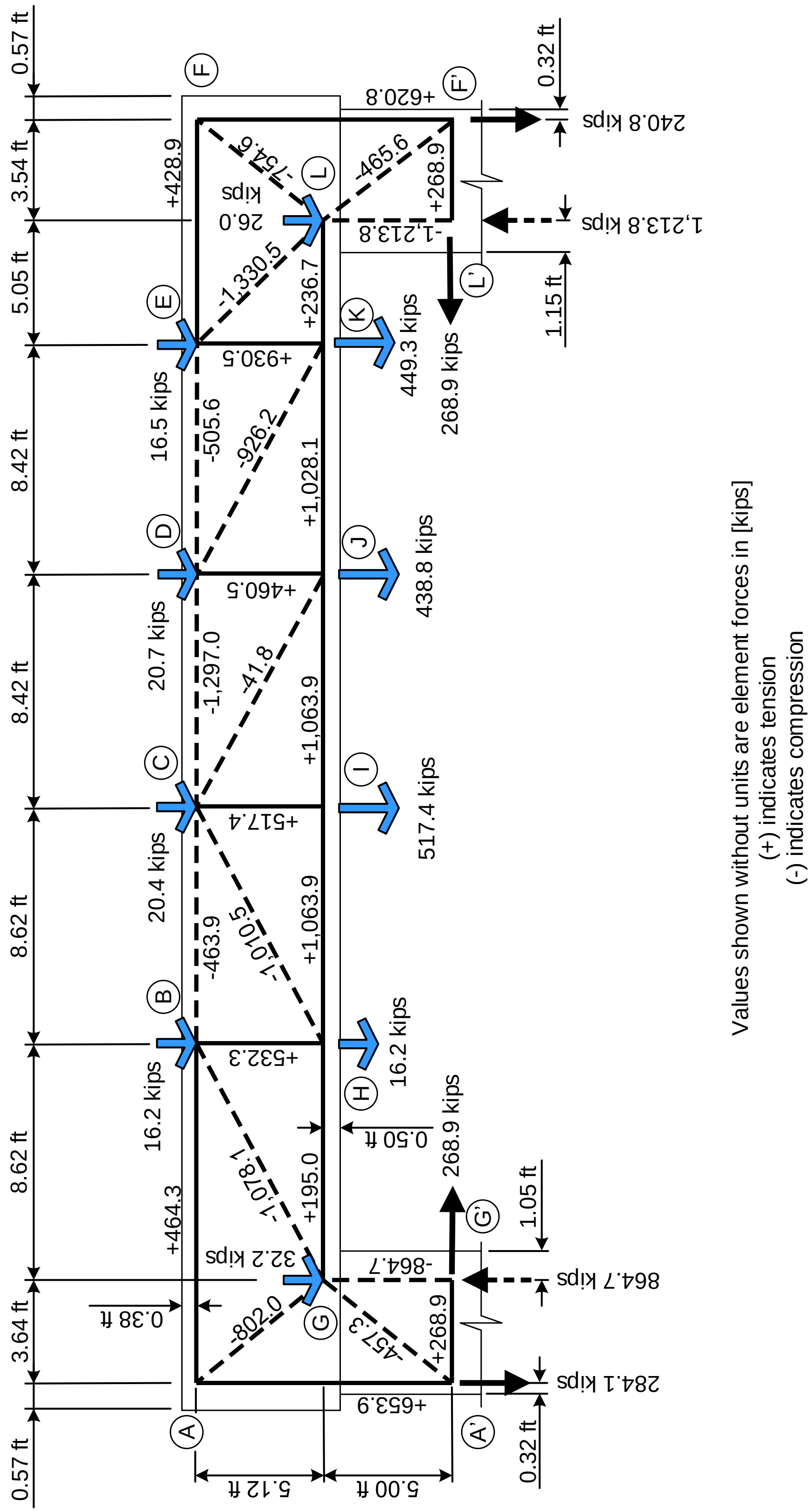
Design Example 3 – Inverted-Tee Moment Frame Straddle Bent Cap:



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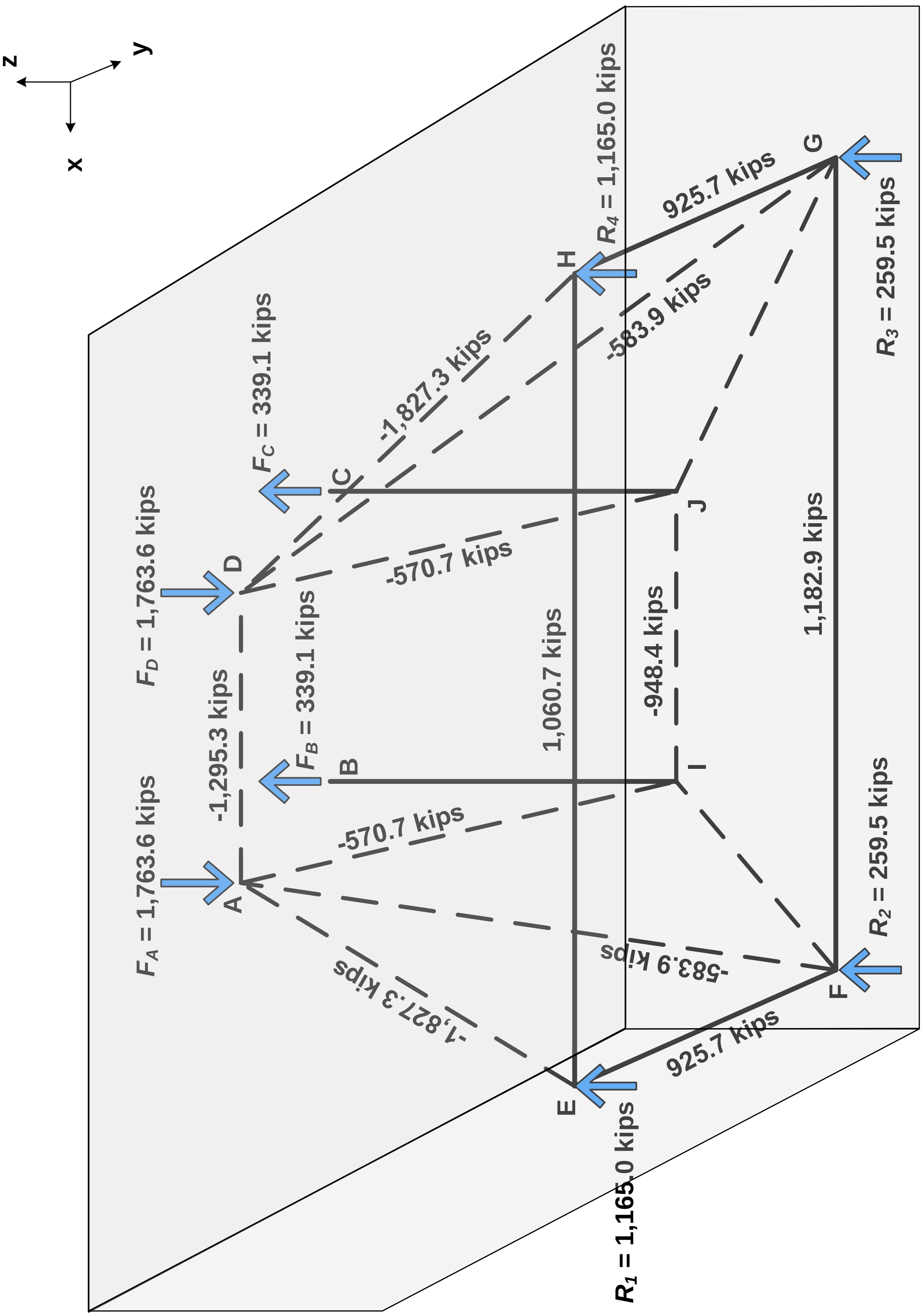
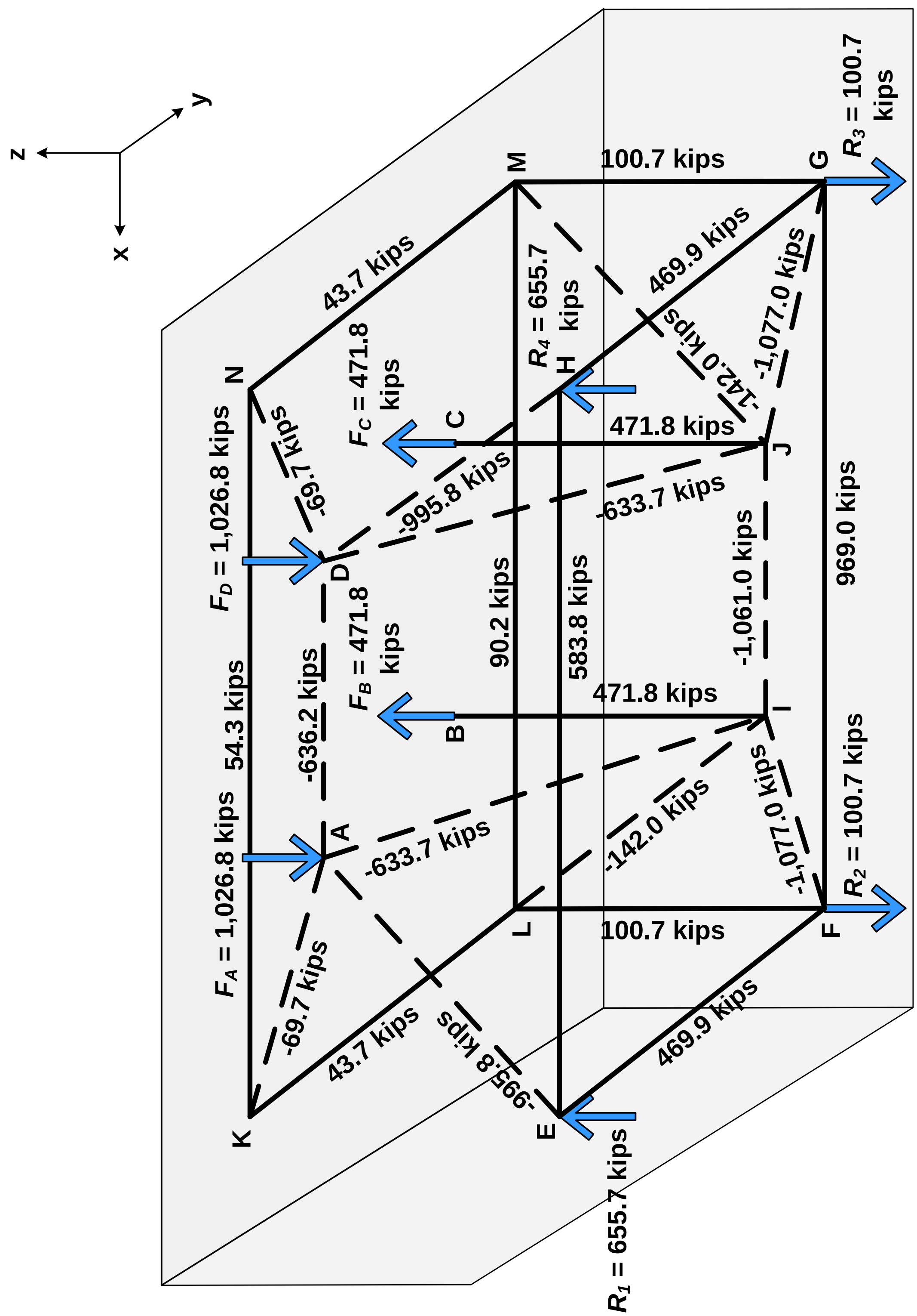
Design Example 4 – Drilled Shaft Footing:

For Load Case 1:



For Load Case 2:



Design Example #1 – Simply-Supported Deep Beam

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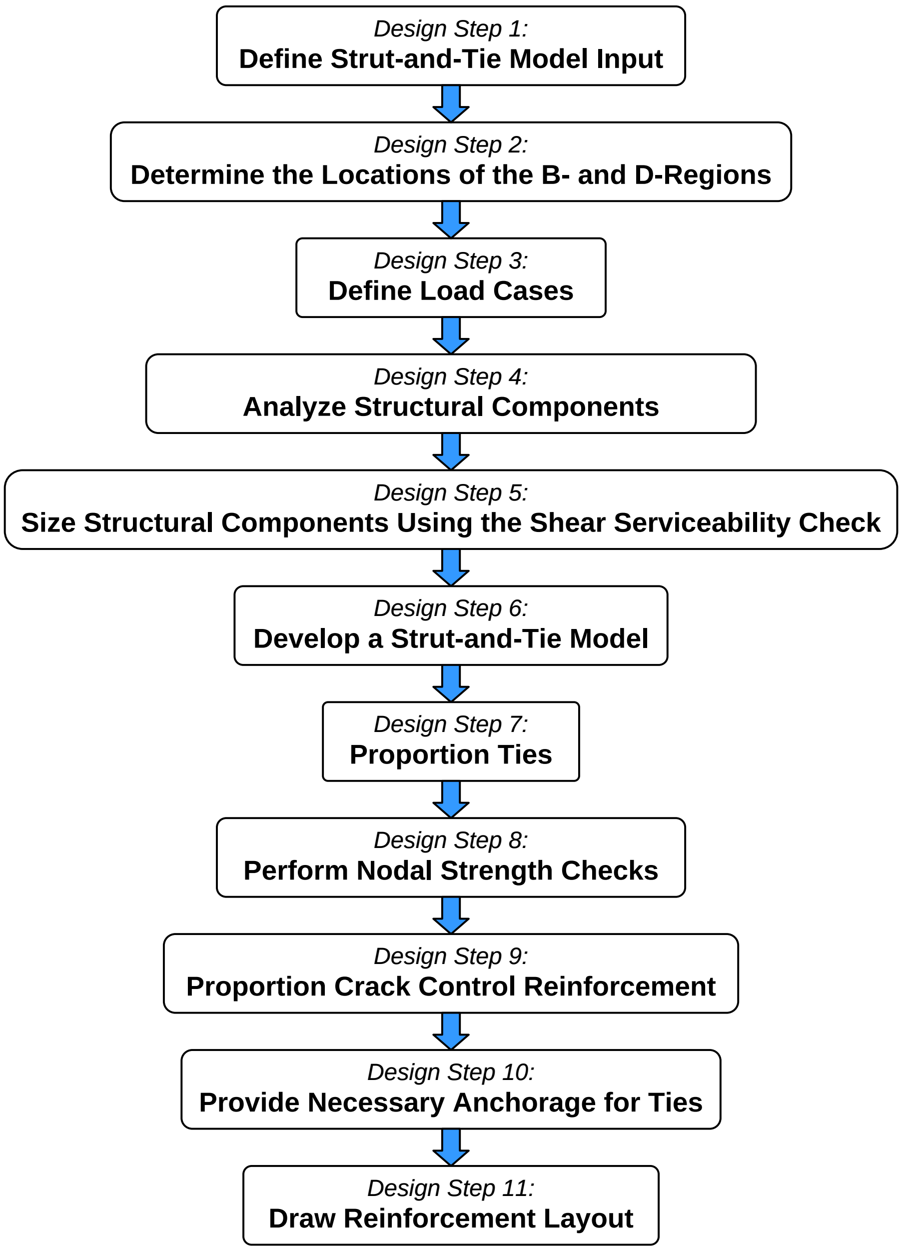
Design Example #1 is one of the classic problems used to demonstrate the application of the strut-and-tie method (STM) to the analysis and design of concrete members. This example demonstrates the sizing, analysis, and design/code checking of a deep beam supporting two concentrated loads. Although this does not represent a specific member in a bridge, a real world analogy to this example would be a straddle bent cap spanning another roadway at a skewed crossing. It also has features similar to the design of a deep pile cap supporting drilled shafts. The example features the elements of strut-and-tie design of concrete members listed below:



Design Step 1 - Define Strut-and-Tie Model Input

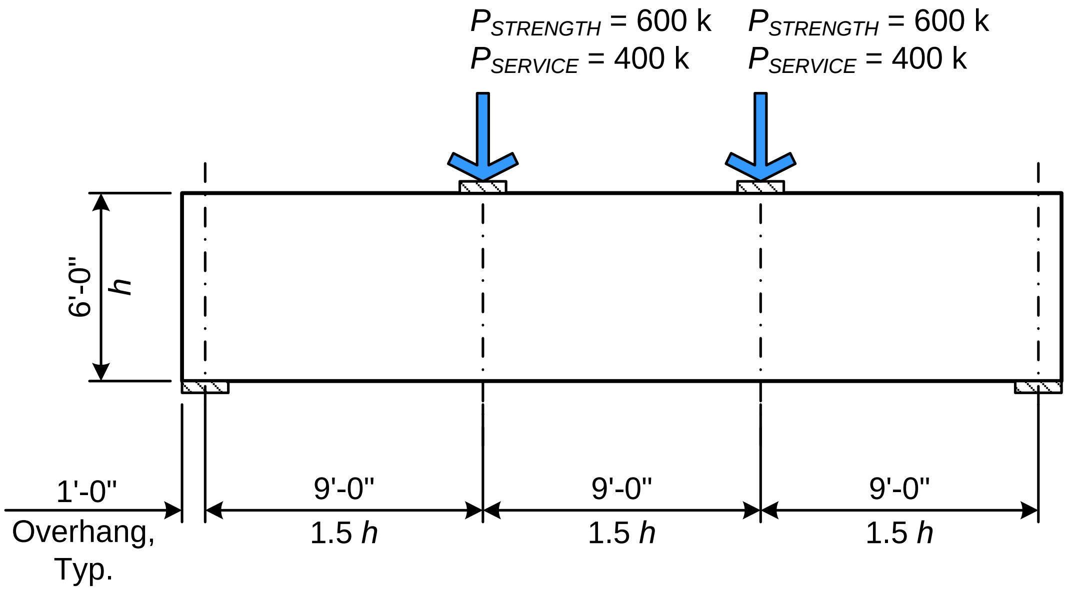


Figure 1-1: Beam Defined for Design Example 1

The beam to be used in this design example is a simply-supported beam which resists two equal concentrated loads on its top face and is supported by bearings at each end (refer to Figure 1-1). The compressive strength for design, f'_c, is taken to be 5.0 ksi, and the yield strength of the steel reinforcing, f_y, is taken as 60 ksi. The overall depth of the beam, h, is assumed to be 6 ft (72 in), and the beam width is assumed to be 4 ft (48 in). The two loads are located at 1.5 times the overall depth of the beam from the centerline of bearing (1.5h). The overall beam span is 27 ft.

Design Step 2 - Determine the Locations of the B- and D-Regions

The definitions of B- and D-Regions are given in *AASHTO LRFD* Article 5.5.1.2. Because it is assumed that all regions within d of a concentrated load qualify as "D-Regions", the entire length of this beam is governed by the strut-and-tie design method. To reinforce the concept of the designation of D-Regions, refer to *AASHTO LRFD* Figure 5.5.1.2.1-1, reproduced on the following page as Figure 1-2.

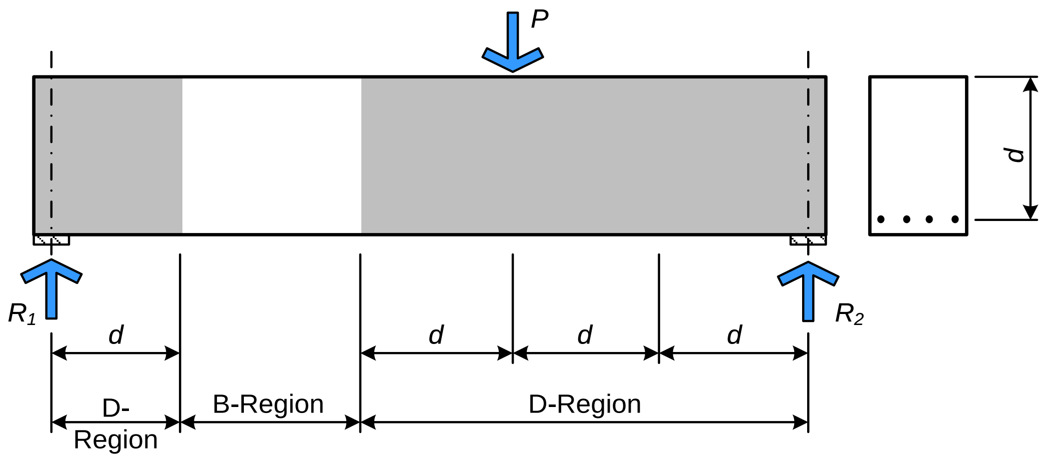


Figure 1-2: Definition of B- and D-Regions

In Figure 1-2, *d* is taken as the effective depth of the member, or the distance from the extreme compression fiber to the centroid of the tension steel. Note that the B-Region of this member is the area more than *d* away from a disturbance (load, reaction, etc.). In this example, for a member 6 ft deep, the distance *d* will be nearly the full height of the member, i.e., *d* and *h* are essentially the same.

Design Step 3 - Define Load Cases

Two concentrated loads are applied to the top surface of the beam. They are assumed to be a combination of dead load and live load, and include the self-weight of the beam. The total service load, such as could come from the *AASHTO LRFD* Service I load combination, is 400 kips. The factored strength load, such as from the Strength I load combination, is 600 kips. The loads are assumed to be point loads distributed to the top of the beam by bearing plates. These are assumed equal to the width of the beam (48 in) and 12 in long parallel to the member.

Design Step 4 - Analyze Structural Components

Starting first with the statics of the problem, it is shown that each reaction is equal to one of the applied point loads. The service and strength load combination forces and reactions are as follows:

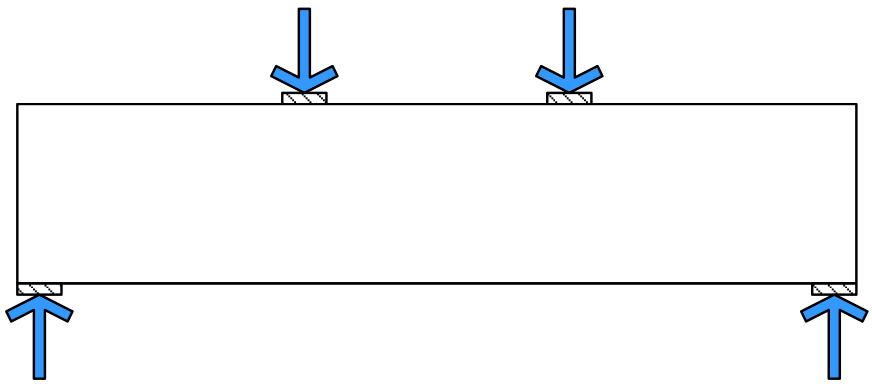


Figure 1-3: Beam Applied Loads and Reactions

In strut-and-tie modeling, traditional (or beam theory) flexure and shear are not the mechanisms of internal load distribution. However, in Design Step 5, the moments determined through traditional beam theory equations at the strength limit state will be used as a tool in establishing the strut-and-tie truss geometry.

Design Step 5 - Size Structural Components Using the Shear Serviceability Check

Design of the simply-supported beam begins with the selection of member dimensions that can be reasonably reinforced. Because the intent of the example is to demonstrate the use of the strut-and-tie method to design D-Regions, the geometry of the example beam, applied loads, and supports is chosen such that the entire beam is governed by D-Regions (i.e., there are no locations along the beam where Bernoulli beam theory applies). Recall that the beam height is 72 in and the beam width is 48 in.

AASHTO LRFD Equation C5.8.2.2-1 limits the applied shear to the following value, V_{cr}, with corresponding minimum and maximum values:

$$V_{cr} = \left[0.2 - 0.1\left(\frac{a}{d}\right)\right]\sqrt{f_c'}b_w d$$

limited as follows:

$$0.0632\sqrt{f_c'}b_w d \ \leq \ V_{cr} \ \leq 0.158\sqrt{f_c'}b_w d$$

where:

b_w = width of the member's web, in
d = effective depth of the member, in

This equation estimates the shear at which diagonal cracks form in D-Regions. Where the applied service load shears are less than V_{cr}, reasonable assurance is provided that diagonal shear cracks will not form. Since this check is performed at the service limit state, the calculated cracking shear will be compared against the service load shear force of 400 kips.

A value for *d* must be assumed at this point. Although it is technically not correct, the LRFD equations for beam-theory flexural strength will be used to determine a trial value of *d* in order to determine V_{cr}. The required flexural strength is a function of the estimated maximum moment. This is found using simple beam statics and the geometry, loads, and reactions shown in Figures 1-1 and 1-3.

$$M_u = P_u x$$

$$x = 1.5h = 1.5 \times 6\ ft \times 12\,\frac{in}{ft} = 108\ in$$

$$M_u = 600\ kips \times 108\ in\ = 64{,}800\ kip - in$$

To begin, it is assumed that a 2 in bottom cover is provided, No. 5 stirrups are used, and two layers of reinforcement will be required (equally divided between the two layers), and a 2 in clear space is provided between layers. If No. 10 reinforcing bars are used, the trial value of *d* is:

$$d = 72\ in - 2\ in - 0.63\ in - 1.27\ in - \frac{2\ in}{2} = 67.1\ in$$

Next, determine the required area of reinforcing, A_s, using *AASHTO LRFD* Equation 5.6.3.2.2-1 (modified by omitting the prestressed reinforcement terms):

$$\phi_f M_n = \phi_f A_s f_y \left(d - {}^{a}/_{2}\right)$$

which is approximated as:

$$\phi_f M_n = \phi_f A_s f_y jd$$

In the equation above, the term *jd* replaces the term *(d - a/2)*. As a “first-pass” approximation, take $j = 0.9$. Using longitudinal reinforcing steel with a yield strength of 60 ksi, $\phi_f = 0.9$, and the assumed values for *d* and *j*, the required area of reinforcing steel is computed as:

$$A_s = \frac{M_u}{\phi_f f_y jd} = \frac{64{,}800\ kip - in}{0.9 \times 60\ ksi \times 0.9 \times 67.1\ in}$$

$$A_s \approx 20\ in^2$$

The area of steel for the tie is intentionally shown as an approximate or rounded number at this point since it is based on a series of approximations. We must now verify if the required area is consistent with the several assumptions of the values of *j* and *d*.

The required area may be provided by 16 No. 10 bars, with 8 in each layer. A sketch of the trial reinforcement layout is provided in Figure 1-4. The sketch allows verification that the width of the beam is adequate and that the reinforcing may be arranged consistently with the initial assumptions of the effective depth, *d*.

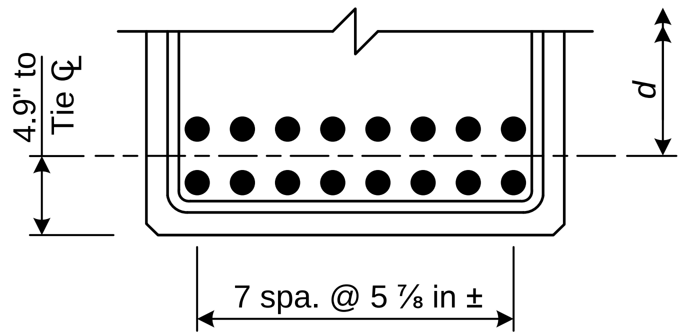


Figure 1-4: Assumed Tie Location

This layout is consistent with prior assumptions for the effective depth, *d*; namely, the reinforcing bar size and clear spacing between the layers are as assumed. The distance to the center of the two layers is:

$$2\ in + 0.63\ in + 1.27\ in + 1\ in = 4.90\ in$$

Checking against the initial assumption of *d* = 67.1 in:

$$d = 72\ in - 4.90\ in = 67.10\ in \rightarrow \boldsymbol{OK}$$

The value of *d* is rounded to 67 in for simplicity. The distance *a* is known as the *shear span*. It is the distance from the centerline of the concentrated load to the centerline of bearing. In this example, *a* is equal to 1.5*h*, or 108 in.

Now that the values of *a* and *d* are known, the value of $\left[0.2 - 0.1\left(\frac{a}{d}\right)\right]$ is found:

$$\left[0.2 - 0.1\left(\frac{108\ in}{67\ in}\right)\right] = 0.0388 < 0.0632$$

Since the calculated value of 0.0388 is less than the lower bound of 0.0632, the lower bound value is used. The value of V_{cr} may now be calculated:

$$V_{cr} = \left[0.2 - 0.1\left(\frac{a}{d}\right)\right]\sqrt{f'_c}\,b_w d$$

$$V_{cr} = 0.0632 \times \sqrt{5.0\ ksi} \times 48\ in \times 67\ in = 454.5\ kips$$

$$454.5\ kips > 400\ kips \quad \boldsymbol{OK}$$

Thus, diagonal cracking should not be expected at the service load shear of 400 kips. Therefore, the following parameters will be used throughout the rest of this design example:

- Total beam height, h = 6 ft = 72 in
- Beam width, b_w = 48 in
- Effective depth, d = 67 in (rounded)

Design Step 6 - Develop a Strut-and-Tie Model

For this design example, two likely load paths may be chosen to distribute the point loads to the bearings. Each is described below and will be used for one-half of the beam design.

The first load path, termed the *direct strut model*, is shown in Figure 1-5 (*AASHTO LRFD* Figure 5.8.2.2-2). The flow of forces is via a direct strut between the applied load on the top surface and the bearing. This model is valid as long as the angle between the inclined strut and the tie is greater than or equal to 25 degrees.

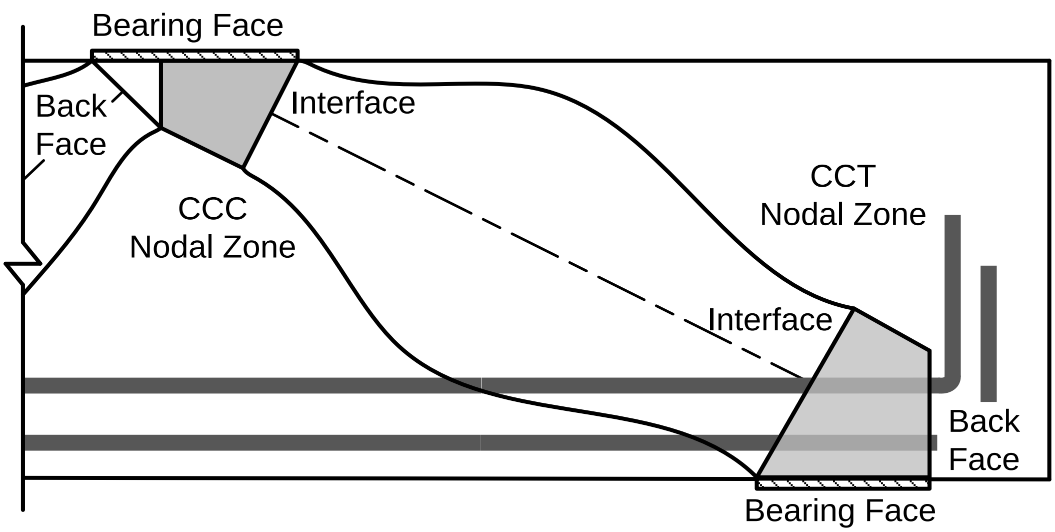


Figure 1-5: Direct Strut-and-Tie Model of a Deep Beam

If, by geometry, the angle between the strut and tie cannot be greater than 25 degrees, one or more intermediate ties may be introduced such that the angle is kept greater than or equal to 25 degrees. This is known as the *two panel model*. The number of vertical ties required is dependent on the ratio of *a/d*, and will increase as the ratio *a/d* increases.

AASHTO LRFD Article C5.8.2.2 states the following regarding the layout of an STM model:

"Minimize the number of vertical ties between a load and a support using the least number of truss panels possible while still satisfying the 25 degree minimum, as shown in Figure C5.8.2.2-3."

> *The 25-degree Limit:*
>
> The goal of the 25-degree limit in *AASHTO LRFD* Article C5.8.2.2 is to preclude excessive strains in the tensile reinforcement in the member. Limiting the tensile strains in the reinforcement limits crack sizes.

When introducing vertical ties, the designer should try to use only the number of ties needed to comply with the 25-degree limit. Using only the minimum required number of ties required results in an efficient beam design. This is illustrated by *AASHTO LRFD* Figure C5.8.2.2-3, reproduced here as Figure 1-6. Studying Figure 1-6, one will note that on the left side, only a single vertical tie is used. On the right side, three vertical ties are used. Both are statically admissible and valid strut-and-tie models. However, by examination, the truss system using only one vertical tie is more efficient than the three-tie system.

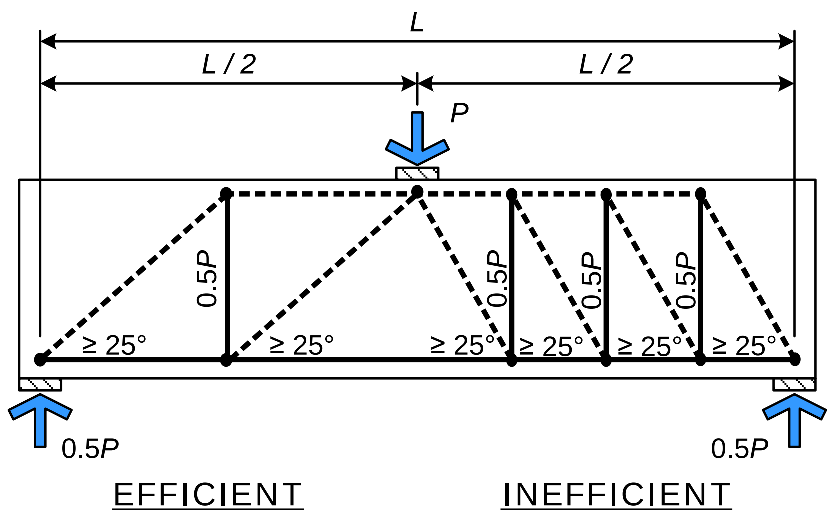


Figure 1-6: Efficient and Inefficient STM Models

By simple statics, it is found that the force in all of the vertical ties is 0.5*P*. In the right-hand truss, the additional ties serve no structural purpose because the resultant force in each tie is not reduced from a single-tie truss. Therefore, the additional vertical ties serve only to increase the quantity of reinforcing steel required, because every tie must be reinforced to support 0.5*P*.

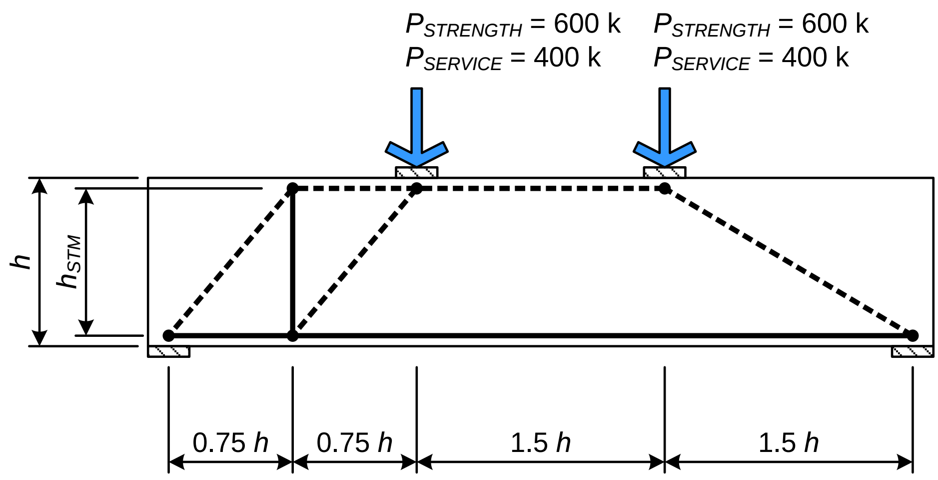


Figure 1-7: Example Beam and Trial Strut-and-Tie Layout

Both the *direct strut model* and the *two panel model* will be used in this design example, each to model one-half of the example beam. The primary purpose of this is to illustrate the differences between the two models, as well as to give example calculations for both methods. The left side point load will be distributed using the two panel model, and the right point load will be distributed using the direct strut model. The example beam and trial strut-and-tie layout are shown in Figure 1-7.

Direct-Strut and Two-Panel Models:

The direct strut model is the most efficient way to model the flow of forces in a strut-and-tie model. Examining Figure 1-7, the reader will note that the left-hand side of the beam may also be modeled using a direct strut. It is modeled as a two-panel model so that calculations for both models may be presented in this example. The angles of the struts for the example beam are shown in Figure 1-8.

The introduction of the vertical tie requires that it must carry the entire vertical force in the truss panel. This may be proven by calculation using the method of sections to solve for the vertical tie force. In the direct strut model, the vertical force is transferred purely by diagonal compression. No stirrups, i.e. vertical ties, would be required other than the crack control reinforcement, which is covered in Design Step 9.

Developing a Strut-and-Tie Model:

Any statically-admissible truss that is in external and internal equilibrium may be used for a strut-and-tie model. However, the reader would be correct in pointing out that the direct strut model used in the right half of the beam is an unstable truss. This type of truss is acceptable to use in strut-and-tie design. Refer to *AASHTO LRFD* Article C5.8.2.2 for additional information.

In order to establish the geometry of the truss, nodes must be located and the vertical distance between the truss chords, h_{STM}, must be determined. The distance h_{STM} is shown in Figure 1-7. Nodes are located at each point load and reaction location. Additionally, for the two panel model, nodes are introduced at each end of a vertical tie.

There are no fixed rules for establishing the height of the truss, h_{STM}, as shown in *AASHTO LRFD* Figure C5.8.2.2-2. It is accepted practice to use a flexural model to determine an approximate height of the truss. Since the effective depth, *d*, was determined in Design Step 5, it will be used in this design step to find the height of the truss. By assuming the center of the tie is located at the centroid of the bottom reinforcing, the depth of the flexural compressive stress block can be found and used to estimate the location of the top horizontal compressive strut.

> *The Strut-and-Tie Model Truss:*
>
> The height of the strut-and-tie model truss may also be found by trial and error by varying the height of the top strut (the top chord of the truss) and checking for equilibrium. The use of beam theory to estimate the height of the strut is an expeditious way to estimate h_{STM}. However, remember that the results of the beam theory calculations for the beam's internal forces are not valid within the D-Regions of the beam and should not be used for any other calculations.

The height of the compression chord is estimated using the traditional Whitney stress block approximation of the depth of the compression zone of a flexural member:

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

Recall that 16 No. 10 bars were assumed in Design Step 5. The area of reinforcing is then:

$$A_s = 16 \times 1.27 \frac{in^2}{bar} = 20.32\ in^2$$

The other design variables have already been defined:

f_y = 60 ksi
f'_c = 5.0 ksi
b = 48 in

Solving for *a*:

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{20.32\ in^2 \times 60\ ksi}{0.85 \times 5.0\ ksi \times 48\ in} = 5.98\ in$$

which is rounded up to 6 in.

The dimension *a* is the assumed height of the top strut. The top nodes are located at the mid-thickness of this dimension, i.e., 3 in from the top surface of the beam. Therefore, the resulting height of the truss, h_{STM}, is:

$$h_{STM} = 72\ in - 5\ in - 3\ in = 64\ in$$

The resulting truss is shown in Figure 1-8. Node designations are given in circles.

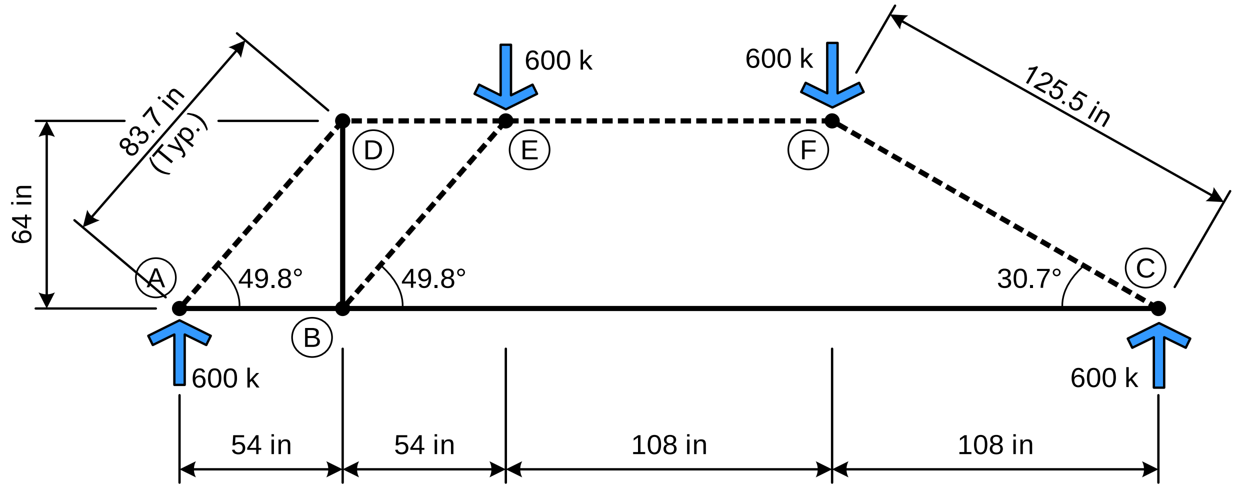


Figure 1-8: Design Example 1 STM Truss

Using the forces at the Strength I load combination (two loads of 600 kips each), the forces in the truss members are found. Calculations for the individual truss element forces are not given here; however, they may be calculated simply by the method of sections or a similar method. A negative (-) sign indicates compression, and a positive (+) sign indicates tension. In the sketch of the truss, it is common practice that struts be drawn as dashed lines and ties be drawn as solid lines. This graphic convention is adhered to throughout this example.

Table 1-1: Strut-and-Tie Model Forces

Node	Member	Force (kips)
A	AB	+506
A	AD	-785
B	AB	+506
B	BC	+1,013
B	BD	+600
B	BE	-785
C	BC	+1,013
C	CF	-1,177
D	AD	-785
D	BD	+600
D	DE	-506
E	BE	-785
E	DE	-506
E	EF	-1,013
F	CF	-1,177
F	EF	-1,013

Design Step 7 - Proportion Ties

Now that the forces in all of the truss members have been determined, the verification of the ties, struts, and nodes will begin. This example begins with the verification of the bottom tie and then the vertical ties, or stirrups.

Proportion the Bottom Tie:

From the truss analysis, the maximum force in the bottom chord tie is in member *BC*, a force of 1,013 kips. The assumed reinforcing pattern developed previously includes two rows of 8 No. 10 reinforcing bars.

AASHTO LRFD Article 5.8.2.4.1 provides the tie strength requirements. The nominal resistance of a tie is given by *AASHTO LRFD* Equation 5.8.2.4.1-1:

$$P_n = f_y A_{st} + A_{ps}\left[f_{pe} + f_y\right]$$

Strength of Ties:

For strut-and-tie models containing no prestressed reinforcement, the prestressed reinforcement terms in Equation 5.8.2.4.1-1 may be taken as zero. Where there is no non-prestressed reinforcement, the term f_y may be taken as 60 ksi in the second term of Equation 5.8.2.4.1-1, but the sum of f_{pe} and f_y shall not be greater than the yield strength of the prestressing steel. The goal of this equation is to limit the stress in the prestressing steel to a value less than or equal to its yield strength, which aids in limiting cracking.

For a strut-and-tie model, the resistance factor, ϕ, is given by *AASHTO LRFD* Article 5.5.4.2 as follows:

- For compression in strut-and-tie models 0.70
- For tension in strut-and-tie models:
  - o Reinforced concrete.. 0.90
  - o Prestressed concrete... 1.00

Thus for a tie, ϕ = 0.90.

Check the factored strength of the tie using the area of reinforcement determined previously. The factored load to be resisted is 1,013 kips.

$$\phi P_n = \phi f_y A_{st} = 0.9 \times 60\ ksi \times 20.32\ in^2 = 1{,}097\ kips$$

$$1{,}097\ kips > 1{,}013\ kips \quad \boldsymbol{OK}$$

Since $\phi P_n > P_u$, the bottom tie design is acceptable.

Proportion the Vertical Tie:

In order to proportion the vertical tie, the amount and placement of the reinforcing that comprise the tie must be determined. The tie connects two “interior nodes,” that is, nodes not acted on by direct loads or bounded by bearing plates. *AASHTO LRFD* Article C5.8.2.2 indicates that a check of the stresses at such nodes is “unnecessary,” but the tie must still be proportioned and detailed accordingly.

Figure 1-9 suggests a method of locating the vertical steel corresponding to the fan-shaped strut. Although the strut-and-tie truss model implies that struts and ties occupy a specific location, in reality the stresses spread out over portions of the member as shown in *AASHTO LRFD* Figure C5.8.2.2-2, reproduced here as Figure 1-9. *AASHTO LRFD* suggests that an “available length” be determined over which the stirrups can be distributed. It is both inefficient and unwise to concentrate all of the reinforcing steel exactly where the theoretical tie is located. By spreading the vertical steel out, the individual bars better resist the overall distribution of forces as the concentrated load from the reaction and applied load fan out across the web depth and length.

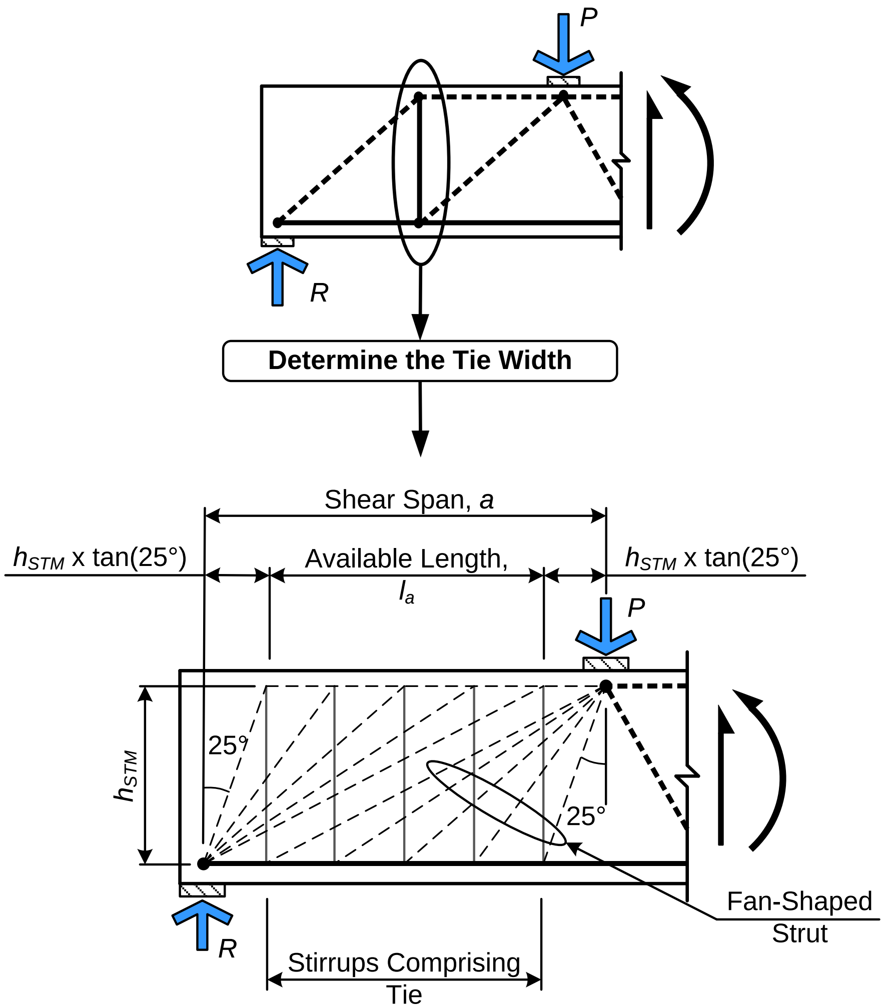


Figure 1-9: Fan-Shaped Struts Engaging Reinforcement, Forming a Tie

The reader is encouraged to refer back to the discussion in Design Step 6 to review the 25-degree limit from which Figure 1-9 is developed. Using Figure 1-9 as a guide, Figure 1-10 is generated to determine the available length over which to distribute the vertical steel, l_a.

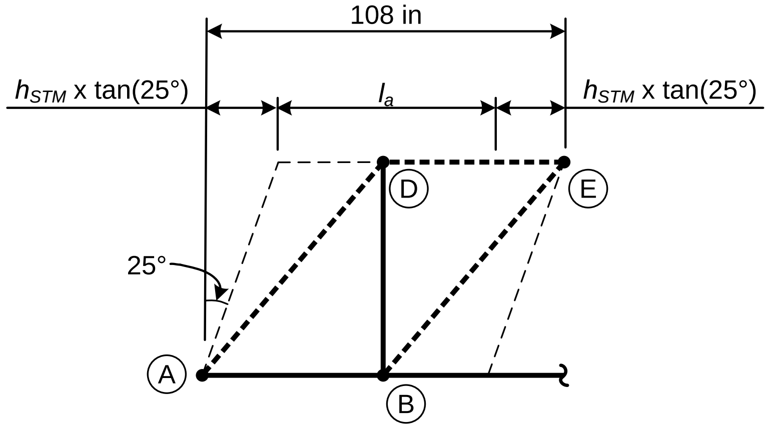


Figure 1-10: Design Example Fan-Shaped Strut

For this design example, the following variables have been calculated previously and are used to determine the available length:

- Shear span, *a* = 108 in
- h_{STM} = 64 in

Therefore:

$$h_{STM} \times \tan 25° = 64\ in \times \tan 25° = 29.8\ in$$

which is rounded up to 30 in. Therefore, the available length, l_a, is given by:

$$l_a = 108\ in - 30\ in - 30in = 48\ in$$

In the strut-and-tie truss model, the force in the vertical tie is in member *BD*. From Table 1-1, the force in the member *BD* is found to be 600 kips. The required area of reinforcing steel may be found by setting *AASHTO LRFD* Equation 5.8.2.4.1-1 equal to 600 kips and solving for A_{st}:

$$\phi P_n = \ \phi f_y A_{st}$$

$$A_{st} \geq \frac{\phi P_n}{\phi f_y} = \frac{600\ kips}{0.9 \times 60\ ksi} = 11.11\ in^2$$

Thus, a minimum of 11.11 in^2 of reinforcing steel must be placed within the available length, l_a. Try 9 sets of 2 No. 5 stirrups (a total of 4 legs per stirrup):

$$A_{st,provided} = 9 \times 2\ stirrups \times 2\ legs \times 0.31 \frac{in^2}{leg} = 11.16\ in^2$$

$$11.16\ in^2 > 11.11\ in^2 \quad \boldsymbol{OK}$$

Check the center-to-center spacing of the individual stirrups against the minimum spacing of 1.5 in given by *AASHTO LRFD* Article 5.10.3.1:

$$s = \frac{48.0\ in}{8\ spaces} = 6.0\ in > 1.5\ in \quad \boldsymbol{OK}$$

The vertical tie reinforcement layout is shown in Figure 1-11.

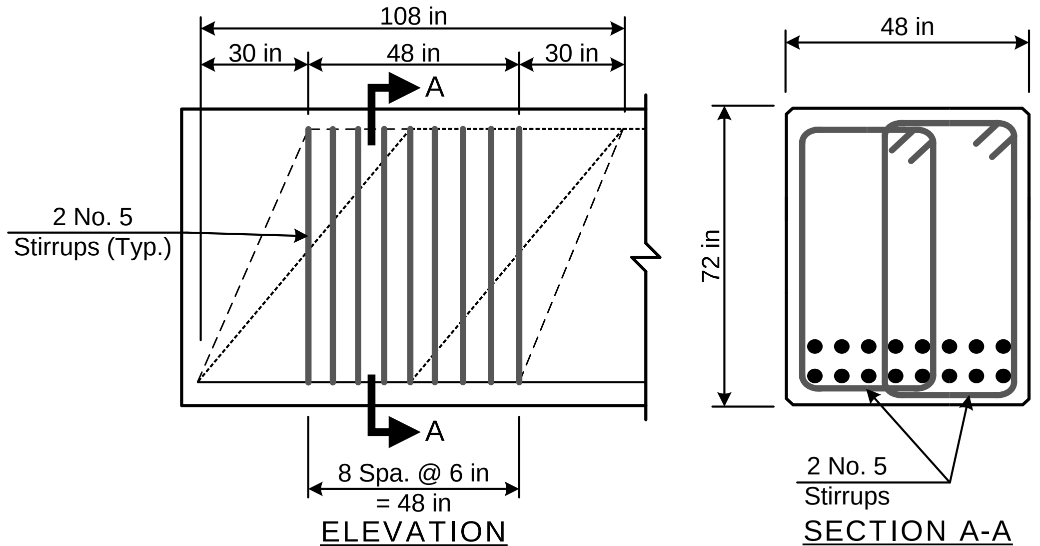


Figure 1-11: Vertical Tie Reinforcing Layout

Design Step 8 - Perform Nodal Strength Checks

Next, the various nodes must be proportioned and checked for adequate strength. This step begins by examining the node at the right bearing, Node *C*.

> *Types of Nodes:*
>
> Nodes may be characterized as CCC, CCT, or CTT nodes. CCC nodes (Compression-Compression-Compression) are nodes where only struts intersect. CCT nodes (Compression-Compression-Tension) are nodes where a tie intersects the node in only one direction. CTT (Compression-Tension-Tension) nodes are nodes where ties intersect a node in two different directions.

***Check Node* C**:

The forces acting at Node *C* are shown in Figure 1-12. There are three forces intersecting at this node: two compressive forces and one tensile force; therefore, this is a CCT node. Two rows of reinforcing steel and an assumed 12 in by 48 in bearing plate, which supports the 600 kip reaction force, are shown:

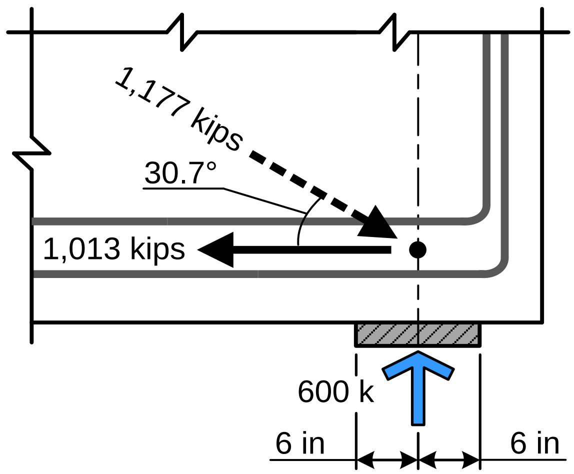


Figure 1-12: Forces Acting on Node *C*

The geometry of a CCT node is given in *AASHTO LRFD* Figure 5.8.2.2-1(b), reproduced here as Figure 1-13:

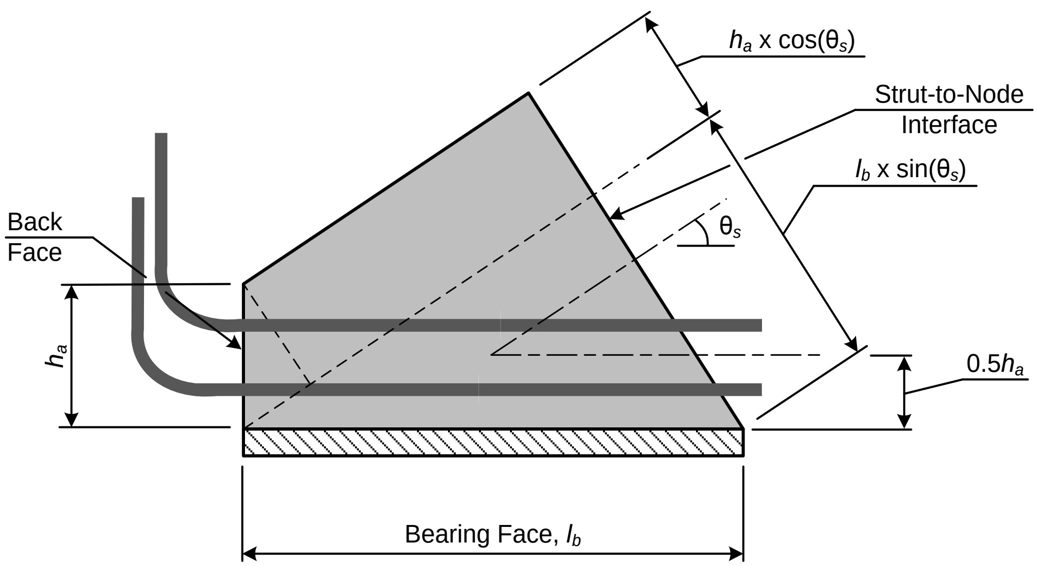


Figure 1-13: Geometry of a CCT Node

The variables in Figure 1-13 are defined below:

- Height of the back face of the CCT node, h_a
- Length of the bearing face, l_b
- Angle between a strut and longitudinal axis of the member, θ_s

These variables may be defined for this example using previously-calculated values and the geometries given in Figure 1-12 and Figure 1-13. The dimension of the bearing plate is assumed at this point to be 12 in; therefore, l_b = 12 in. The height of the back face of the CCT node is taken as the height of the tie. This is assumed to be twice the depth from the bottom fiber of the beam to the centroid of the tie reinforcing, per *AASHTO LRFD* Article 5.8.2.5.2. Consequently, h_a is taken as a rounded dimension of 10 in.

The width of the strut-to-node interface, herein called w, is given by:

$$w = l_b \sin\theta_s + h_a \cos\theta_s$$

$$w = 12\ in \times \sin 30.7° + 10\ in \times \cos 30.7° = 14.7\ in$$

The resistance of each node face must be checked against the factored loads on each face. The nominal resistance of a node face is given by *AASHTO LRFD* Equation 5.8.2.5.1-1:

$$P_n = f_{cu} A_{cn}$$

where:

P_n = nominal resistance of a node face, kips
A_{cn} = effective cross-sectional area of the node face, in^2
f_{cu} = limiting compressive stress at the node face, ksi

The value of A_{cn} is determined according to *AASHTO LRFD* Article 5.8.2.5.2. The depth of an individual node face is determined according to *AASHTO LRFD* Figure 5.8.2.2-1. The out-of-plane dimension may be determined by the bearing device dimensions or the width of member, as appropriate.

The value of f_{cu} is determined according to *AASHTO LRFD* Article 5.8.2.5.3. The value of f_{cu} is given by Equation 5.8.2.5.3a-1:

$$f_{cu} = m v f'_c$$

where:

f'_c = compressive strength of concrete used in design, ksi
m = confinement modification factor (defined below)
v = concrete efficiency factor

The confinement modification factor, m, is defined by:

$$m = \sqrt{\frac{A_2}{A_1}} \leq 2.0$$

where:

A_1 = area under the bearing device, in^2
A_2 = notional area, defined by *AASHTO LRFD* Article 5.6.5, in^2

Notional Area, A_2*:*

The stress on the face of a node is assumed to be uniformly distributed. Where a supporting surface is larger than the loaded area, area A_2 is calculated assuming that the load spreads out from the loaded area at a rate of 2H:1V, until the edge of a member is met. Otherwise, if the loaded area is equal to the total member area, *m* is taken as 1.0. The areas A_1 and A_2 are illustrated in AASHTO LRFD Figure 5.6.5-1, reproduced below as Figure 1-14.

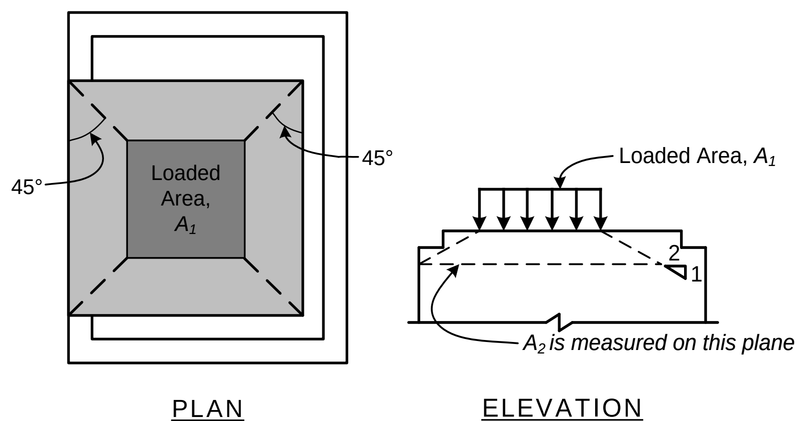


Figure 1-14: Determining the Areas A_1 and A_2

The concrete efficiency factor, *v*, is dependent on the presence of crack control reinforcement in the member under consideration. If crack control reinforcement is not present (as specified by *AASHTO LRFD* Article 5.8.2.6), *v* shall be taken as 0.45.

For structures that do contain crack control reinforcement defined by Article 5.8.2.6, *v* is determined from *AASHTO LRFD* Table 5.8.2.5.3a-1, reproduced below as Table 1-2. Calculations for satisfying the crack control reinforcement requirement will be shown in Design Step 9.

Table 1-2: Concrete Efficiency Factors, *v*

Face	CCC Node	CCT Node	CTT Node
Bearing Face	0.85	0.70	$0.85 - \frac{f'_c}{20\ ksi}$ $0.45 \leq \nu \leq 0.65$
Back Face	0.85	0.70	$0.85 - \frac{f'_c}{20\ ksi}$ $0.45 \leq \nu \leq 0.65$
Strut-to-Node Interface	$0.85 - \frac{f'_c}{20\ ksi}$ $0.45 \leq \nu \leq 0.65$	$0.85 - \frac{f'_c}{20\ ksi}$ $0.45 \leq \nu \leq 0.65$	$0.85 - \frac{f'_c}{20\ ksi}$ $0.45 \leq \nu \leq 0.65$

Recall that Node *C* is a CCT node. Therefore, for the bearing face:

$$A_{cn} = 12\ in\ \times 48\ in = 576\ in^2$$

$$v = 0.70$$

And for the strut-to-node interface:

$$A_{cn} = 14.7\ in\ \times 48\ in = 706\ in^2$$

$$v = 0.85 - \frac{f'_c}{20\ ksi} = 0.85 - \frac{5\ ksi}{20\ ksi} = 0.60$$

For both node faces, since the strut is the full width/thickness of the member, the confinement modification factor, *m*, equals 1.0.

The resistance factor, ϕ, for compression in a strut-and-tie model is found in *AASHTO LRFD* Article 5.5.4.2, which is equal to 0.70. Thus, the factored resistance of a nodal face in a strut-and-tie model is calculated by *AASHTO LRFD* Equation 5.8.2.5.1-1, modified as shown:

$$\phi P_n = \phi m v f'_c A_{cn}$$

For the bearing face:

$$\phi P_n = 0.7 \times 1.0 \times 0.7 \times 5.0\ ksi \times 576\ in^2 = 1{,}411\ kips$$

$$1{,}411\ kips > 600\ kips \quad \boldsymbol{OK}$$

For the strut-to-node interface:

$$\phi P_n = 0.7 \times 1.0 \times 0.6 \times 5.0\ ksi \times 706\ in^2 = 1{,}482\ kips$$

$$1{,}482\ kips > 1{,}177\ kips \quad \boldsymbol{OK}$$

A check of the back face is not required. Bond stresses from reinforcing steel development need not be applied to the back face of a CCT node, per *AASHTO LRFD* Article 5.8.2.5.3b. If the bars were anchored with headed reinforcing or an anchor plate, the stresses on the back face of the node would be checked.

The Back Face of a CCT Node:

AASHTO LRFD Article C5.8.2.5.3b states that there were no experimental cases reviewed where the back face stress of a CCT node controlled the strength of that node when the tie was composed of deformed reinforcing steel.

However, this is not the case if the stress on the back face is applied by a bearing/anchor plate or a headed bar. If the tie is composed of a headed bar or is anchored by a bearing plate, the back face of the node should be checked assuming that the bar or tendon is unbonded and that all of the tie force is transferred through the anchor plate or bar head.

Check Node* A*:

Because the reactions at Nodes *A* and *C* are the same and the bearings are assumed to be the same size, the check of the bearing surface is satisfied automatically. The diagonal compressive load in member *AD* is significantly less than member *CF* (785 kips vs. 1,177 kips), and by geometry the width of the strut-to-node interface is larger at Node *A* than at Node *C*. Therefore, Node *A* is adequate by inspection.

Check Node* F*:

The geometry of Node *F* and loads acting on the node faces are given in Figure 1-15. The upper bearing plate is assumed to be 12 in long and 48 in wide (similar to the lower bearing plates). The height of the horizontal strut loading the left side of the node, h_a, is the assumed strut depth found in Design Step 6 using the flexural analogy, which is 6 in. This strut is in equilibrium with the tie in the bottom of the beam.

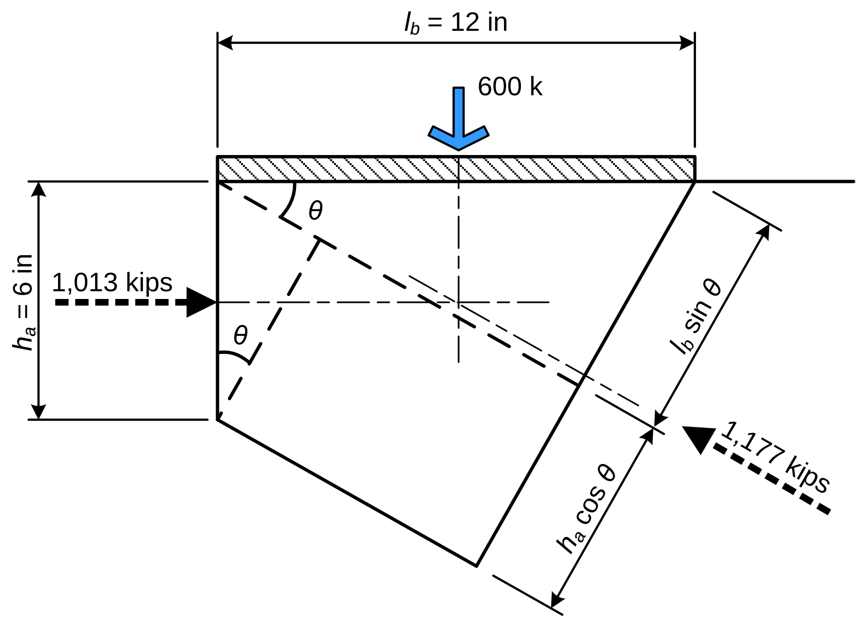


Figure 1-15: Forces Acting on Node *F*

Because all of the forces acting on this node are compressive, this is a CCC node. The geometry of a CCC node is given in *AASHTO LRFD* Figure 5.8.2.2-1(a), reproduced here as Figure 1-16:

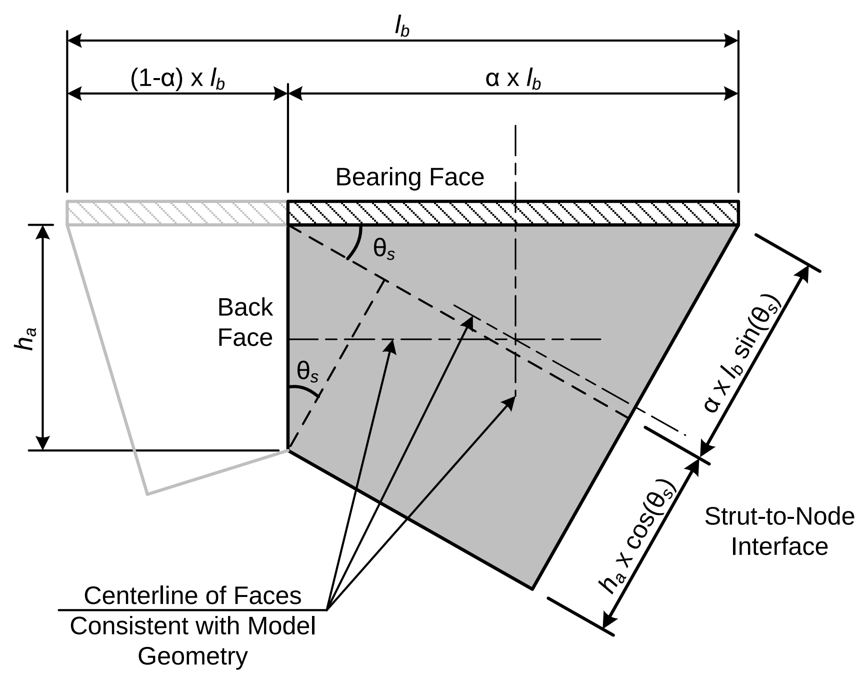


Figure 1-16: Geometry of a CCC Node

The width of the strut at the node interface may be found as for Node *C*:

$$w = l_b \sin\theta_s + h_a \cos\theta_s$$

$$w = 12\ in \times \sin 30.7° + 6\ in \times \cos 30.7° = 11.3\ in$$

CCC Nodes and the Factor **α***:*

In CCC nodes, where diagonal struts enter a node from both sides and an external load *P* is applied, it is sometimes beneficial to separate the applied load into two statically equivalent loads, P_1 and P_2, such that $P_1 + P_2 = P$. These two loads are then assumed to act in the center of the tributary area of the bearing plate. The factor α denotes the portion of the load supported by the right diagonal strut, and (1 - α) denotes the load carried by the left diagonal strut.

This approximation is not used in this example as there is only one diagonal strut, therefore, referring to Figure 1-16, it is assumed that α = 1, and the full length of the bearing plate l_b is used. Refer to *AASHTO LRFD* Article C5.8.2.2 and Figure C5.8.2.2-4 for additional information.

Next, calculate the effective cross-sectional area of each node face, A_{cn}, and determine the concrete efficiency factors, *v*. Recall that the concrete efficiency factors are found in *AASHTO LRFD* Table 5.8.2.5.3a-1 and in Table 1-2 of this design example. The concrete efficiency factors for CCC nodes are illustrated in *AASHTO LRFD* Figure C5.8.2.5.3a-1(a), reproduced below as Figure 1-17:

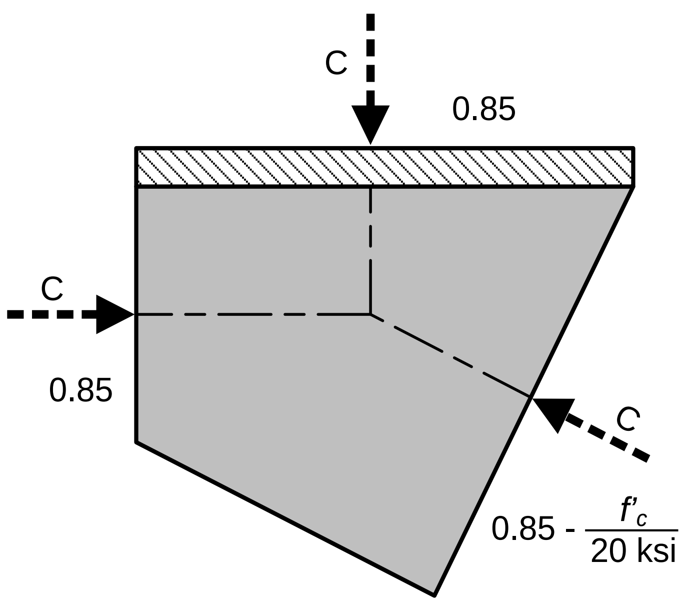


Figure 1-17: Concrete Efficiency Factors for CCC Nodes

For the bearing face:

$$A_{cn} = 12\ in \times 48\ in = 576\ in^2$$

$$v = 0.85$$

For the back (left) face:

$$A_{cn} = 6\ in \times 48\ in = 288\ in^2$$

$$v = 0.85$$

For the inclined strut face:

$$A_{cn} = 11.3\ in \times 48\ in = 542\ in^2$$

$$v = 0.85 - \frac{f'_c}{20\ ksi} = 0.85 - \frac{5.0\ ksi}{20\ ksi} = 0.60$$

Next, using these calculated values, determine the factored resistance of each node face using *AASHTO LRFD* Equations 5.8.2.5.1-1 and 5.8.2.5.3a-1. For both node faces, since the width of the strut is equal to the full width/thickness of the members, the confinement modification factor, m, equals 1.0. Recall that $\phi = 0.7$ for compression in strut-and-tie models. The strength is then calculated by *AASHTO LRFD* Equation 5.8.2.5.1-1:

$$\phi P_n = \phi m v f'_c A_{cn}$$

For the bearing face:

$$\phi P_n = 0.7 \times 1.0 \times 0.85 \times 5.0\ ksi \times 576\ in^2 = 1{,}713\ kips$$

$$1{,}713\ kips > 600\ kips \quad \boldsymbol{OK}$$

For the back (left) face:

$$\phi P_n = 0.7 \times 1.0 \times 0.85 \times 5.0\ ksi \times 288\ in^2 = 856\ kips$$

$$856\ kips < 1{,}013\ kips \quad \boldsymbol{NO\ GOOD}$$

For the inclined strut face:

$$\phi P_n = 0.7 \times 1.0 \times 0.60 \times 5.0\ ksi \times 542\ in^2 = 1{,}138\ kips$$

$$1{,}138\ kips < 1{,}177\ kips \quad \boldsymbol{NO\ GOOD}$$

Two of the checks at this node do not meet the requirements of the specification.

First, in order to increase the strength for the inclined strut, the bearing plate length is increased to 14 in. This increases the strut-to-node interface width, w, to 12.3 in, resulting in a new effective cross-sectional area of 590 in^2. This results in a factored compressive resistance of 1,239 kips, which is greater than 1,177 kips. Consequently, this node is now sufficient. For consistency, all of the bearing plates will be increased to 14 inches in length.

For the horizontal top strut, originally sized to meet the assumed flexural analogy (from Bernoulli beam theory), the strut does not have sufficient resistance. In this example, it is decided to reinforce the top strut. Since some longitudinal mild steel is required anyway (for crack control and to anchor the vertical stirrups) it is quite simple to add steel reinforcement to increase the node face's factored resistance.

Increasing the Strength of a Node's Face:

The four options for increasing the strength of the deficient node face presented below vary in difficulty and time required to implement. They are listed in decreasing order of difficulty:

- Change the truss geometry: Changing the geometry of the STM truss results in a new truss analysis. All of the truss forces must then be recalculated and all tie- and nodal-strength checks must be re-performed, which can be very time-consuming for all but the simplest models.
- Increase the concrete strength: Increasing the concrete strength requires recalculation of the concrete efficiency factors, *v*, and subsequent verification that all calculated node strengths are still sufficient.
- Increase the beam width: Similar to increasing the concrete strength, increasing the beam width requires another iteration of STM design checks. It will also require recalculating the beam self-weight loads.
- Reinforce the struts: Reinforcing the strut only requires calculation of the area of reinforcing steel required to make up the deficiency in resistance. This option is often the simplest.

The top strut is deficient in resistance by:

$$1{,}013\ kips - 856\ kips = 157\ kips$$

Because the strut is a compressive member, the provisions of *AASHTO LRFD* Article 5.6.4.4 may be applied. The area of reinforcing steel required is found by applying *AASHTO LRFD* Equation 5.6.4.4-3, modified by including only the mild steel reinforcement terms (since the resistance of the concrete strut has already been determined and there is no prestressed reinforcement):

$$\phi P_n = \phi \times 0.80\left[f_y A_{st}\right]$$

Rearranging and solving for A_{st}:

$$A_{st} \geq \frac{\phi P_n}{0.80\phi f_y} = \frac{157\ kips}{0.80 \times 0.70 \times 60\ ksi} = 4.67\ in^2$$

The area of reinforcing steel required to satisfy the strength requirement at this node is 4.67 in^2. Try providing 6 No. 8 bars in the strut:

$$A_{st,provided} = 6 \times 0.79 \frac{in^2}{bar} = 4.74\ in^2$$

$$4.74\ in^2 > 4.67\ in^2 \quad \boldsymbol{OK}$$

Therefore, provide 6 No. 8 reinforcing bars near the top of the beam.

Check Node* E*:

At Node *E*, multiple forces intersect at the same location; there is a load applied at a bearing and three internal truss member loads at this node. The strut-and-tie design method, as defined by experiment and adopted by AASHTO, was developed by assuming three forces intersect each node. Therefore, nodes where there are more than three intersecting loads must have some forces combined into resultant forces. This also results in somewhat simpler computations.

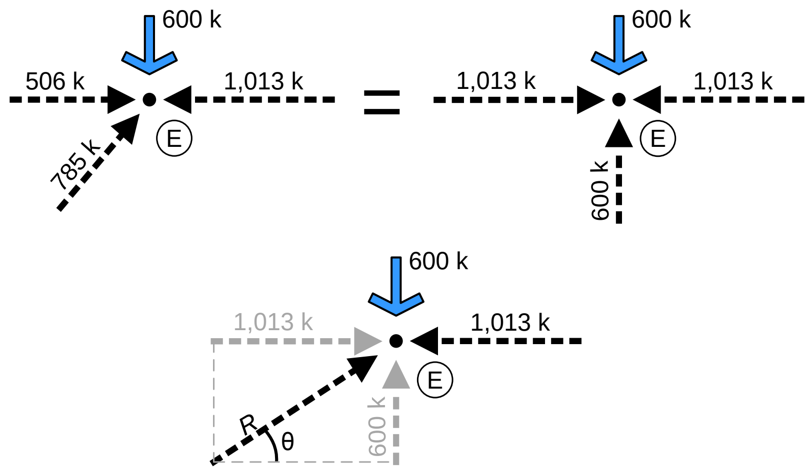


Figure 1-18: Combining Nodal Forces into Resultants

The top-left portion of Figure 1-18 depicts the loads at Node *E* as they are drawn in the strut-and-tie model. Depicted is the 600-kip applied load and the resultant strut forces. The top-right portion of Figure 1-18 shows the forces at Node *E* with the 785-kip diagonal load separated into its horizontal and vertical components. Note that all of the forces acting on Node *E* remain in equilibrium.

The bottom portion of Figure 1-18 shows a resultant force, *R*, determined by combining the force in member *DE* and the horizontal and vertical components of force in member *BE.* The resultant force, *R*, and the angle of its line of action, θ, are found by:

$$R = \sqrt{F_H^2 + F_V^2} = \sqrt{(1{,}013\ kips)^2 + (600\ kips)^2} = 1{,}177\ kips$$

$$\theta = \tan^{-1}\left(\frac{Vertical}{Horizontal}\right) = \tan^{-1}\left(\frac{600\ kips}{1{,}013\ kips}\right) = 30.7°$$

The resulting force of *R* = 1,177 kips is the same force as exists, by symmetry, in member *CF* acting at the same angle, θ, and was previously analyzed in the prior check of Node *F*. Because Node *F* was eventually shown to be satisfactory, Node *E* is therefore also satisfactory since it is statically equivalent to the forces at Node *F*.

Static Equivalency:

Recall that the left side of the strut-and-tie model was chosen to be a *two panel model* only to demonstrate the design of a beam with a vertical tie. If it were replaced with a *direct strut model*, it would be identical to the right-hand side of the strut-and-tie model.

Nodes B *and* D*:*

Nodes *B* and *D* are not checked for compressive resistance. As discussed in Design Step 7 when the vertical tie was designed, the tension force within the tie is actually spread out over the available length, l_a. Hence, no check is required.

Smeared Nodes:

Refer to Figure 1-9, which was referenced while designing the vertical tie between Nodes *B* and *D*. Recall that a vertical tie in a strut-and-tie model does not exist in a distinct location; rather, forces spread out over the shear span, *a* (the distance between the applied load and reaction). The nodes at the ends of the vertical tie are referred to as *smeared nodes*.

Smeared nodes do not have a geometry that can be clearly defined by a bearing plate or the boundaries of the member. Therefore, the limits of a nodal region cannot be determined with any degree of certainty. The forces in a smeared node are able to disperse over a large area and thus are less critical than *singular nodes*, or nodes with clearly defined geometries, and checking the concrete stresses at smeared nodes is typically unnecessary.

Design Step 9 - Proportion Crack Control Reinforcement

Crack control reinforcement, provided as orthogonal grids of reinforcing bars, is provided both to limit the width of cracks and to provide a minimum level of ductility, so that if required, inelastic redistribution of stresses can occur. Because the nodal zones of the example beam have been designed using the concrete efficiency factors of *AASHTO LRFD* Table 5.8.2.5.3a-1, crack control reinforcement is required.

The spacing of the crack control reinforcing may not exceed the smaller of $d/4$ or 12.0 in, in both directions.

Controlling Cracks:

Placing crack control reinforcement is a requirement for satisfactory performance of a concrete member under service loads. This reinforcement aids in controlling the width of cracks under service loads, and it also restrains the compressive stress within struts. Restraining the compressive stress helps prevent side-bursting failure of the concrete in the struts.

The crack control reinforcement in the vertical direction shall satisfy *AASHTO LRFD* Equation 5.8.2.6-1:

$$\frac{A_v}{b_w s_v} \geq 0.003$$

The crack control reinforcement in the horizontal direction shall satisfy *AASHTO LRFD* Equation 5.8.2.6-2:

$$\frac{A_h}{b_w s_h} \geq 0.003$$

where:

A_v = area of vertical reinforcement within spacing s_v, in^2
A_h = area of horizontal reinforcement within spacing s_h, in^2
b_w = width of the member's web, in
s_v = spacing of vertical crack control reinforcement, in
s_h = spacing of horizontal crack control reinforcement, in

Thick and Thin Members:

For thinner members, the crack control reinforcement will consist of two mats of reinforcing, one placed near each face of the member. For thicker members, multiple mats of reinforcing, placed throughout the width of the member, may be required in order to satisfy the 0.3% requirement of Equations 5.8.2.6-1 and -2.

AASHTO LRFD Figure C5.8.2.6-1 (reproduced below as Figure 1-19) illustrates the variables of Equations 5.8.2.6-1 and -2. Note the difference in placement of reinforcement of a thin and thick member. The orthogonal grid of reinforcement restrains the concrete in the strut (shown in gray) from failure by bursting outward from the face of the member.

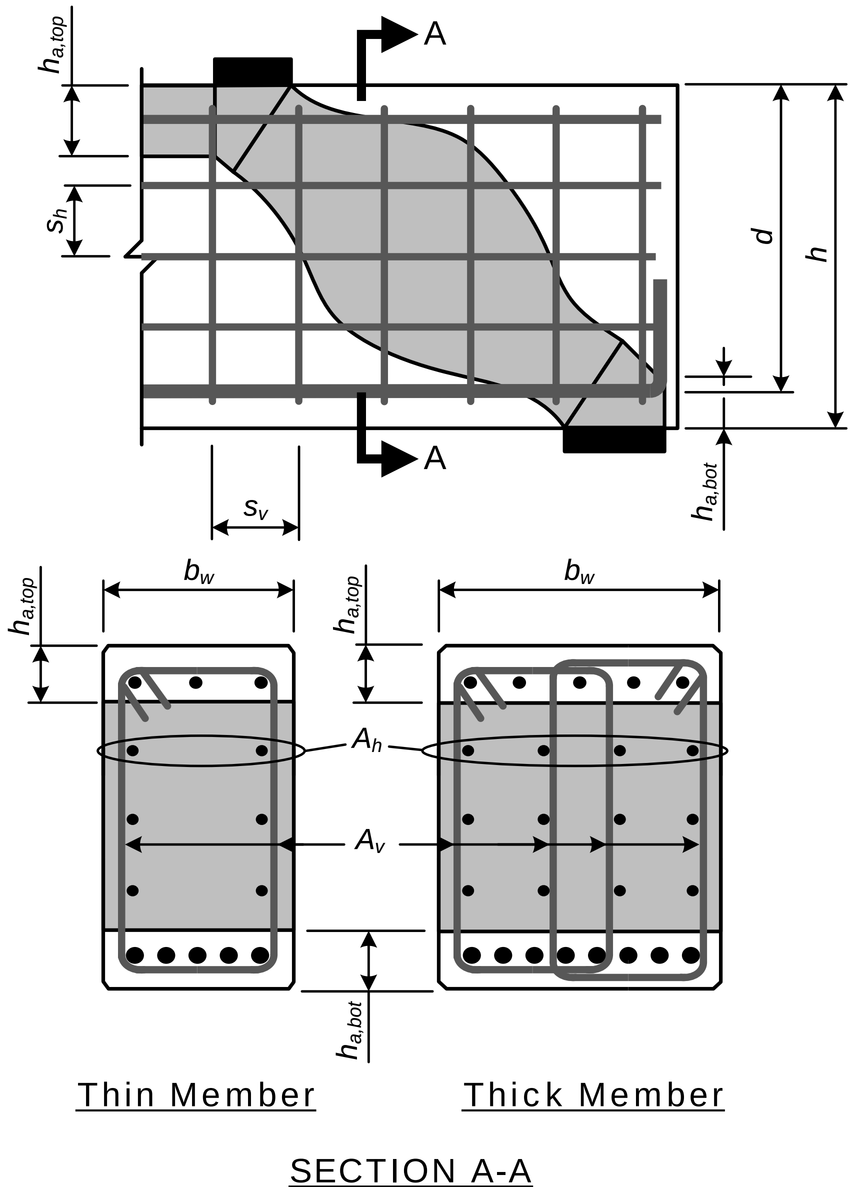


Figure 1-19: Distribution of Crack Control Reinforcement

Solve for the area of reinforcing required in the horizontal direction:

$$\frac{A_h}{s_h} \geq 0.003 b_w = 0.003 \times 48\ in = 0.144\,\frac{in^2}{in} = 1.73\,\frac{in^2}{ft}$$

Try providing 4 No. 6 bars per foot:

$$A_{h,provided} = 4 \times 0.44\,\frac{in^2}{bar} = 1.76\,\frac{in^2}{ft}$$

$$1.76\,\frac{in^2}{ft} > 1.73\,\frac{in^2}{ft} \quad \boldsymbol{OK}$$

Solve for the area of reinforcing required in the vertical direction:

$$\frac{A_v}{s_v} \geq 0.003 b_w = 0.003 \times 48\ in = 0.144\ \frac{in^2}{in} = 1.73\ \frac{in^2}{ft}$$

Try providing 2 No. 5 closed stirrups (4 legs total) spaced at 8 in center-to-center:

$$A_{v,provided} = 2\ stirrups \times 2\ \frac{legs}{stirrup} \times 0.31\ \frac{in^2}{leg} \times \frac{12\ \frac{in}{ft}}{8\ in} = 1.86\ \frac{in^2}{ft}$$

$$1.86\ \frac{in^2}{ft} > 1.73\ \frac{in^2}{ft} \quad \boldsymbol{OK}$$

For the right half of the beam (where the direct strut model is used), this is the only reinforcing steel required. For the left half of the beam, recall that the vertical tie had its own reinforcing steel requirements. That reinforcing requirement was met by 2 No. 5 closed stirrups (4 legs total) at 6 in centers, a tighter spacing than necessary for the direct strut model.

A comparison of the direct strut and two panel models' reinforcement requirements is given in Table 1-3.

Table 1-3: Reinforcement Requirements Comparison

Location	Direct Strut Model	Two Panel Model
Vertical Reinforcing	4 Legs of No. 5 Stirrups at 8 in Centers	4 Legs of No. 5 Stirrups at 8 in Centers Typ., Except 4 Legs of No. 5 Stirrups at 6 in Centers, Centered Around the Vertical Tie
Bottom Chord (Tie) Reinforcing	16 No. 10 Bars	16 No. 10 Bars

Note that the bottom tie reinforcing is independent of the direct strut or two panel truss model. In the region between the point loads, the region of constant moment, the moment in the pseudo-simple beam is independent of the truss configuration and would be the same no matter what discretization is used between the point loads and the reactions.

The comparison in Table 1-3 demonstrates the efficiency of the direct strut model versus the two panel model. Recall that the two panel model was used in this example only to demonstrate its application, and in this example it can be replaced by a direct strut model. The addition of the vertical tie requires providing additional reinforcing to carry the same forces carried by the direct strut model.

Design Step 10 - Provide Necessary Anchorage for Ties

The ends of the hooked No. 10 longitudinal bars are checked for required development. Unlike a flexural model where the reinforcing steel is checked for development at the points of maximum moment, the tie of an STM truss model has a constant force between its end nodes. Consequently, the anchorage of reinforcing steel, particularly near the ends of beams and faces of members, is critical.

Refer to Figure 1-20 below *(AASHTO LRFD* Figure C5.8.2.4.2-1). The reinforcing bars of the ties must be properly anchored to guarantee that the tie force is fully developed and the structure can achieve the resistance calculated in the strut-and-tie model. In order for a tie to be considered properly anchored, the full yield resistance of the tie should be developed at the point where the centroid of the reinforcing steel exits the *extended nodal zone*, as shown.

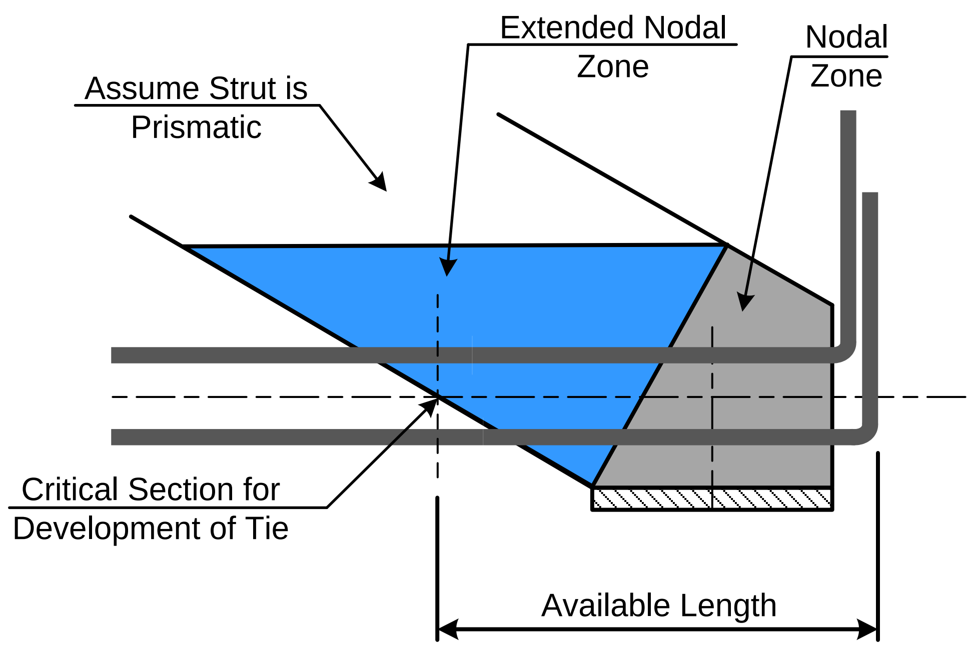


Figure 1-20: Available Development Length for Ties

Critical Section for Development of the Tie:

The location of the *critical section* for the development of the tie reinforcing is shown graphically in Figure 1-20. This point occurs where the centroid of the reinforcing steel in the tie passes through the edge of strut that intersects the tie. Its location may be determined by assuming that the strut is prismatic, where its edges are parallel to its centerline.

The demand in the bottom chord is a maximum in member *BC* and was found to be 1,013 kips. In Design Step 7, the tie capacity was found to be 1,097 kips. Because some excess capacity is present, it may be used to reduce the required development length.

The required development length for a hooked reinforcing bar in tension is given by *AASHTO LRFD* Equation 5.10.8.2.4a-1:

$$l_{dh} = l_{hb} \times \left(\frac{\lambda_{rc}\lambda_{cw}\lambda_{er}}{\lambda}\right)$$

where l_{hb} is given by *AASHTO LRFD* Equation 5.10.8.2.4a-2:

$$l_{hb} = \frac{38d_b}{60} \times \left(\frac{f_y}{\sqrt{f_c'}}\right)$$

where:

λ =	concrete density modification factor
λ_{rc} =	reinforcement confinement factor
λ_{cw} =	reinforcement coating factor
λ_{er} =	excess reinforcement factor

The following values are assumed for calculation:

λ =	1.0 (Normal-weight concrete)
λ_{rc} =	1.0 (Conservative assumption)
λ_{cw} =	1.0 (Uncoated reinforcement, i.e., black bars)

The excess reinforcement factor, λ_{er}, is calculated by:

$$\lambda_{er} = \frac{A_{s,required}}{A_{s,provided}} \propto \frac{P_u}{\phi P_n} = \frac{1{,}013\ kips}{1{,}097\ kips} = 0.92$$

Therefore:

$$l_{hb} = \frac{38 \times 1.27\ in}{60} \times \left(\frac{60\ ksi}{\sqrt{5\ ksi}}\right) = 21.6\ in$$

$$l_{dh} = 21.6\ in \times \left(\frac{1.0 \times 1.0 \times 0.92}{1.0}\right) = 19.9\ in$$

which is rounded up to 20 in.

The available development length is determined graphically in Figure 1-21. Recall that the top bearing plates were lengthened to 14 in during the analysis of Node *F*, and this change was carried through to the bottom bearing plates as well.

Anchorage of the tie reinforcing is checked at the critical section near Node *C*. The distance from the end of the bearing plate to the critical section, *x*, is determined by geometry:

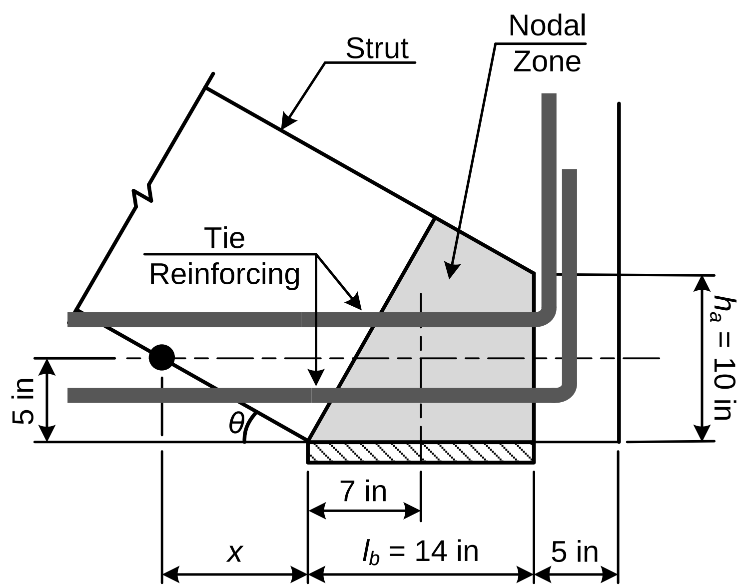


Figure 1-21: Geometry of Tie End Hook Development near Node *C*

$$x = \frac{5\ in}{\tan\theta} = \frac{5\ in}{\tan 30.7°} = 8.4\ in$$

The available development length is determined by:

$$l_{dh,available} = (8.4\ in + 14\ in + 5\ in) - 2\ in\ cover = 25.4\ in$$

$$25.4\ in > 20.0\ in \quad \boldsymbol{OK}$$

Therefore, the development length is adequate, and the tie is capable of developing its required strength.

Design Step 11 - Draw Reinforcement Layout

At this point, the strut-and-tie analysis for the example simply-supported deep beam is complete. Sketches of the final beam dimensions and reinforcing layouts follow.

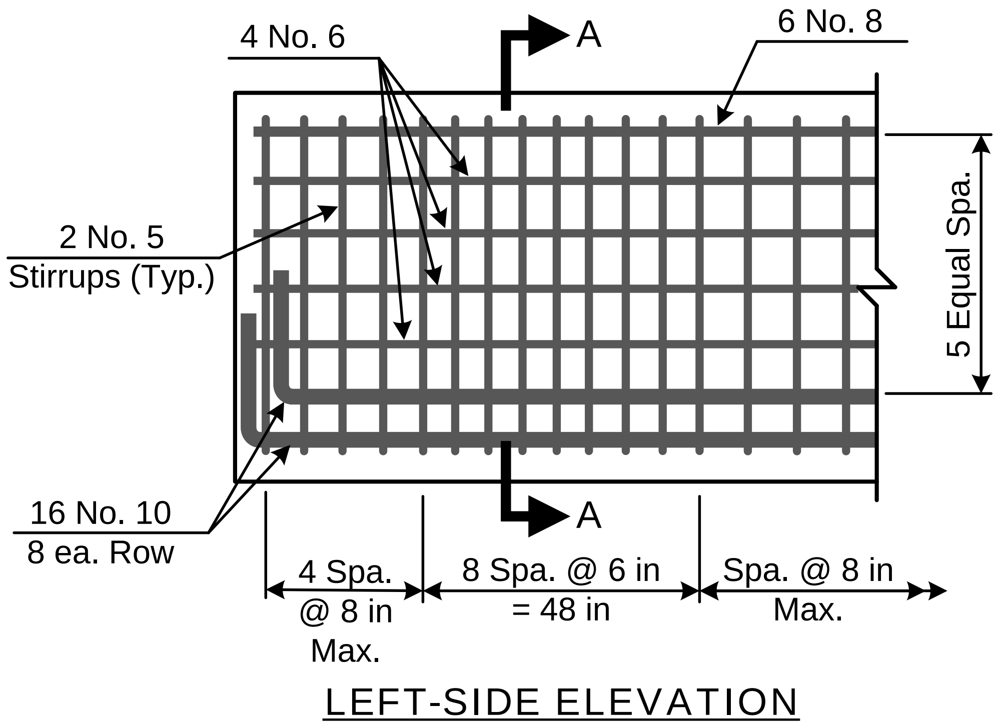


Figure 1-22: Reinforcing Layout, Left Side

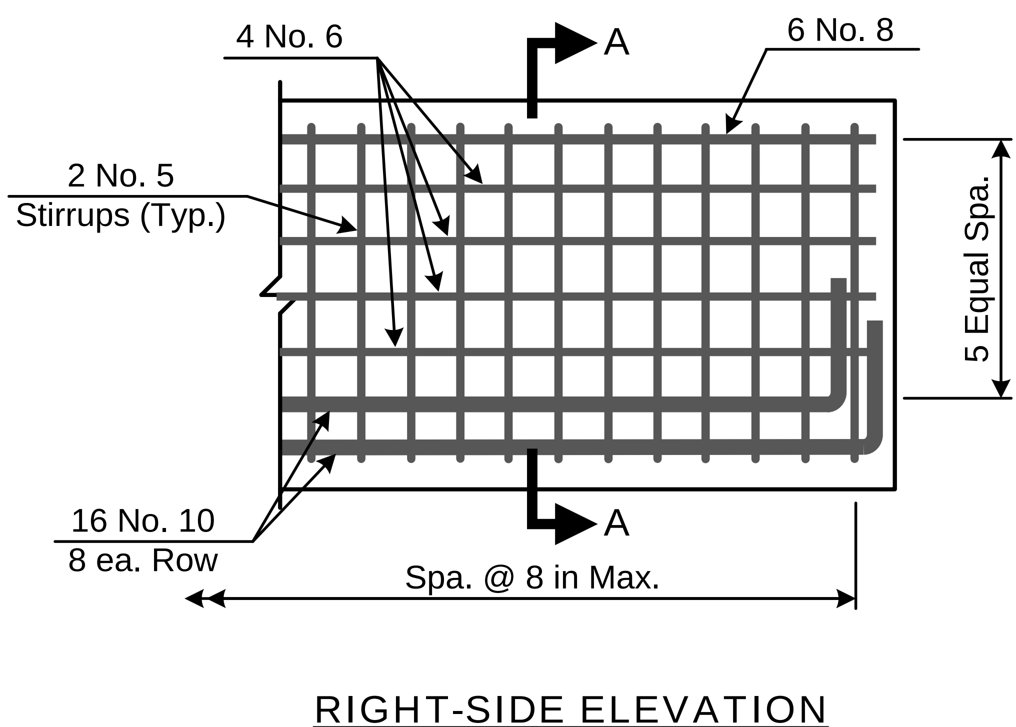


Figure 1-23: Reinforcing Layout, Right Side

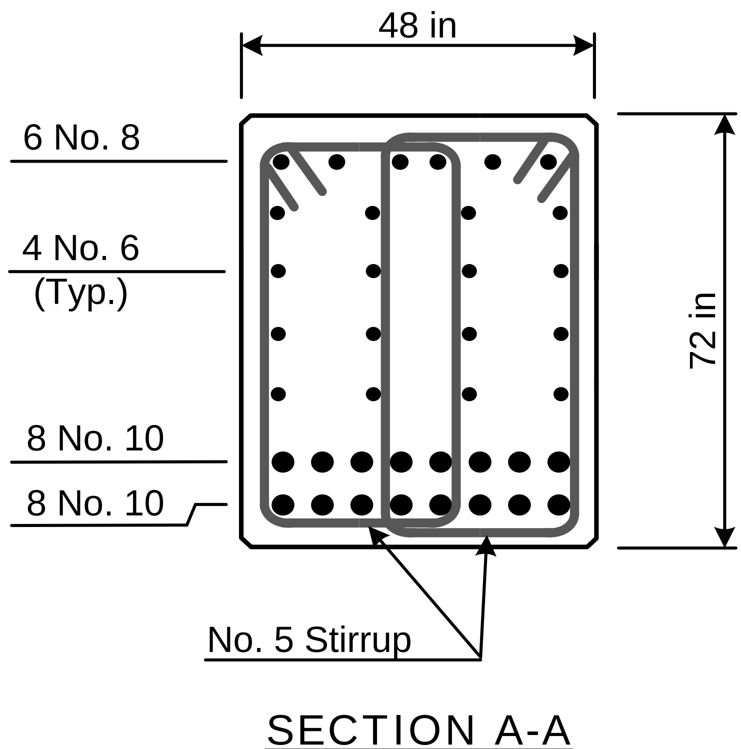


Figure 1-24: Section through Example Beam

Design Example #2 – Cantilever Bent Cap

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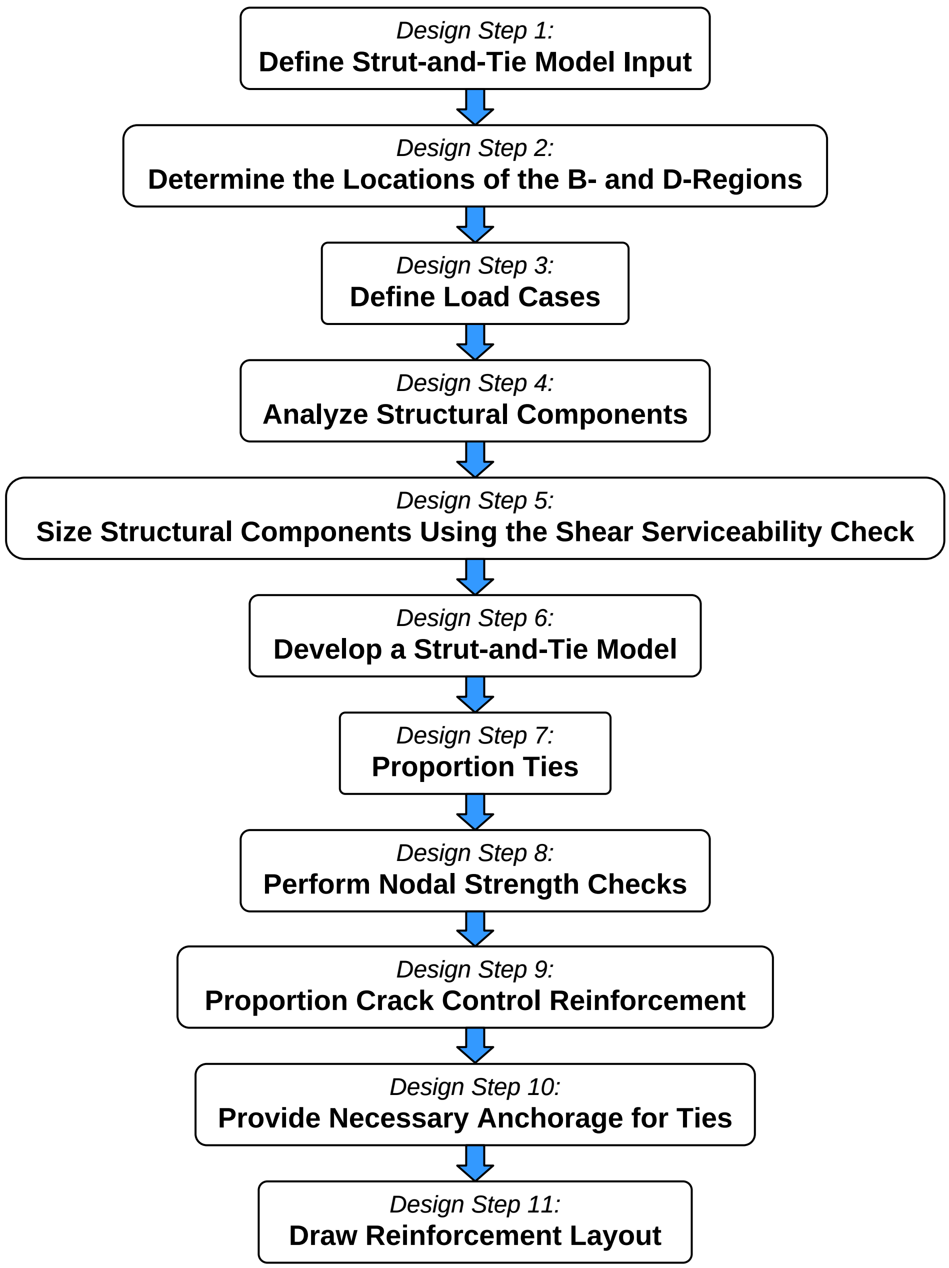
Design Example #2 presents the design of a cantilever bent cap utilizing the strut-and-tie modeling (STM) design procedure. Step-by-step STM procedures are presented, and a complete design is demonstrated for one of the load cases that must be considered. This design example includes developing a strut-and-tie model for a slightly sloped structure. The cantilever bent cap is sloped to accommodate the banked grade of the roadway supported by the bent, and the applied loads are therefore not perpendicular to the primary longitudinal chord of the STM. A flowchart of the various design steps is presented below:



It should be noted that the STM provisions in the 8th edition of *AASHTO LRFD* (2017) are based primarily on research conducted at the University of Texas at Austin (Birrcher et al., 2009). This design example is based primarily on one of the example problems included in an implementation project sponsored by TxDOT (5-5253-01, Williams et al., 2011). In addition, figures in this design example have been adapted from Williams et al. (2011). The example problem originally prepared by Williams et al. (2011) has been revised to provide additional explanation and to be fully compliant with the STM provisions of the 8th edition of *AASHTO LRFD*, as appropriate.

Design Step 1 - Define Strut-and-Tie Model Input

Elevation and plan views of the cantilever bent cap are presented in Figures 2-1 and 2-2. For clarity, a simplified view (excluding bearing pads and bearing seats) is shown in Figure 2-1. However, a more detailed geometry of the cap is presented in Figure 2-2.

For this design example, the cantilever bent cap supports two prestressed concrete U-beams from one direction and two steel girders from the opposite direction. Each of the U-beams rests on two neoprene bearing pads, while each of the steel girders is supported by a single pot bearing. The bearing conditions of each girder are shown in Figure 2-2.

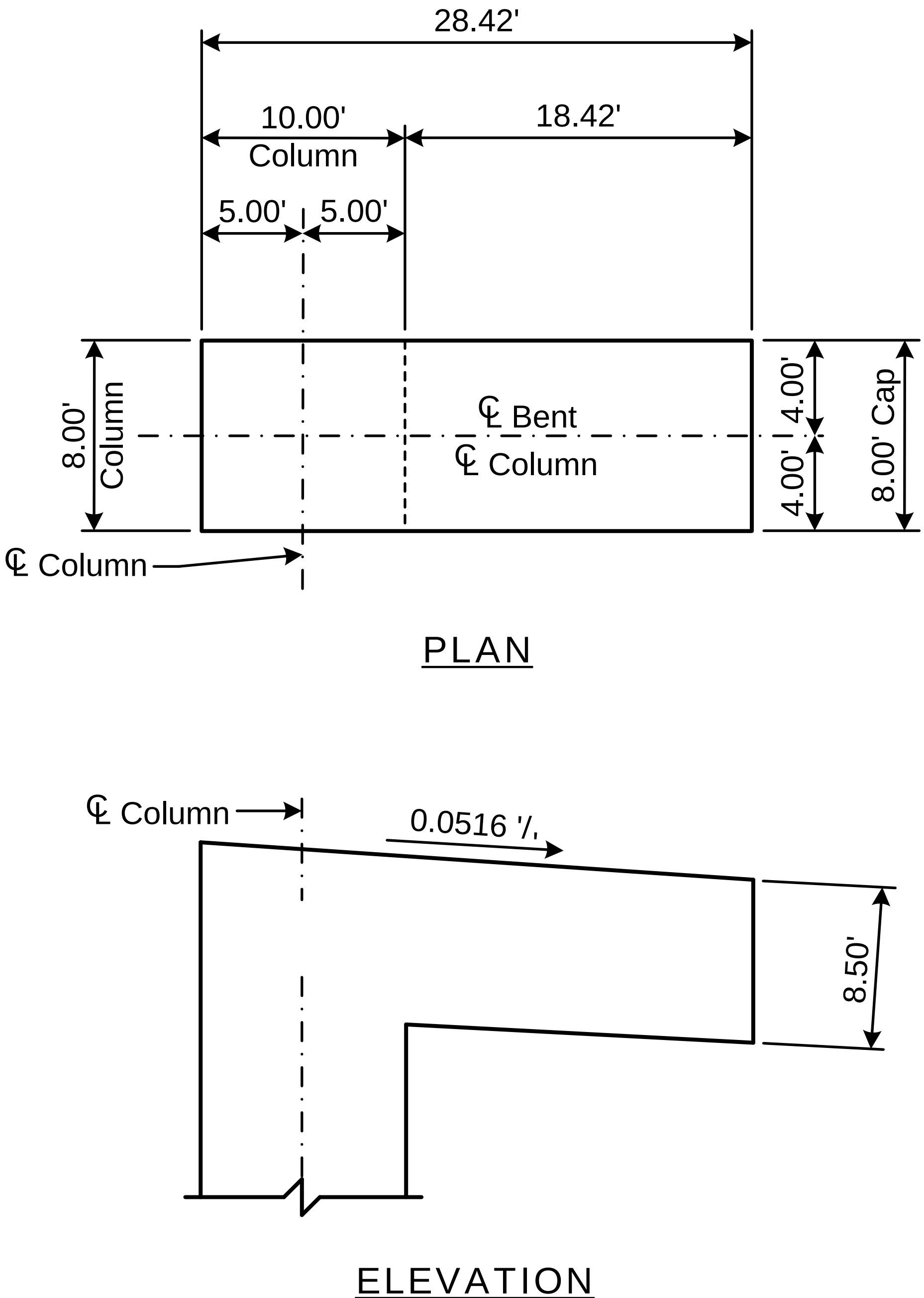


Figure 2-1: Plan and Elevation Views of Cantilever Bent Cap (Simplified Geometry)

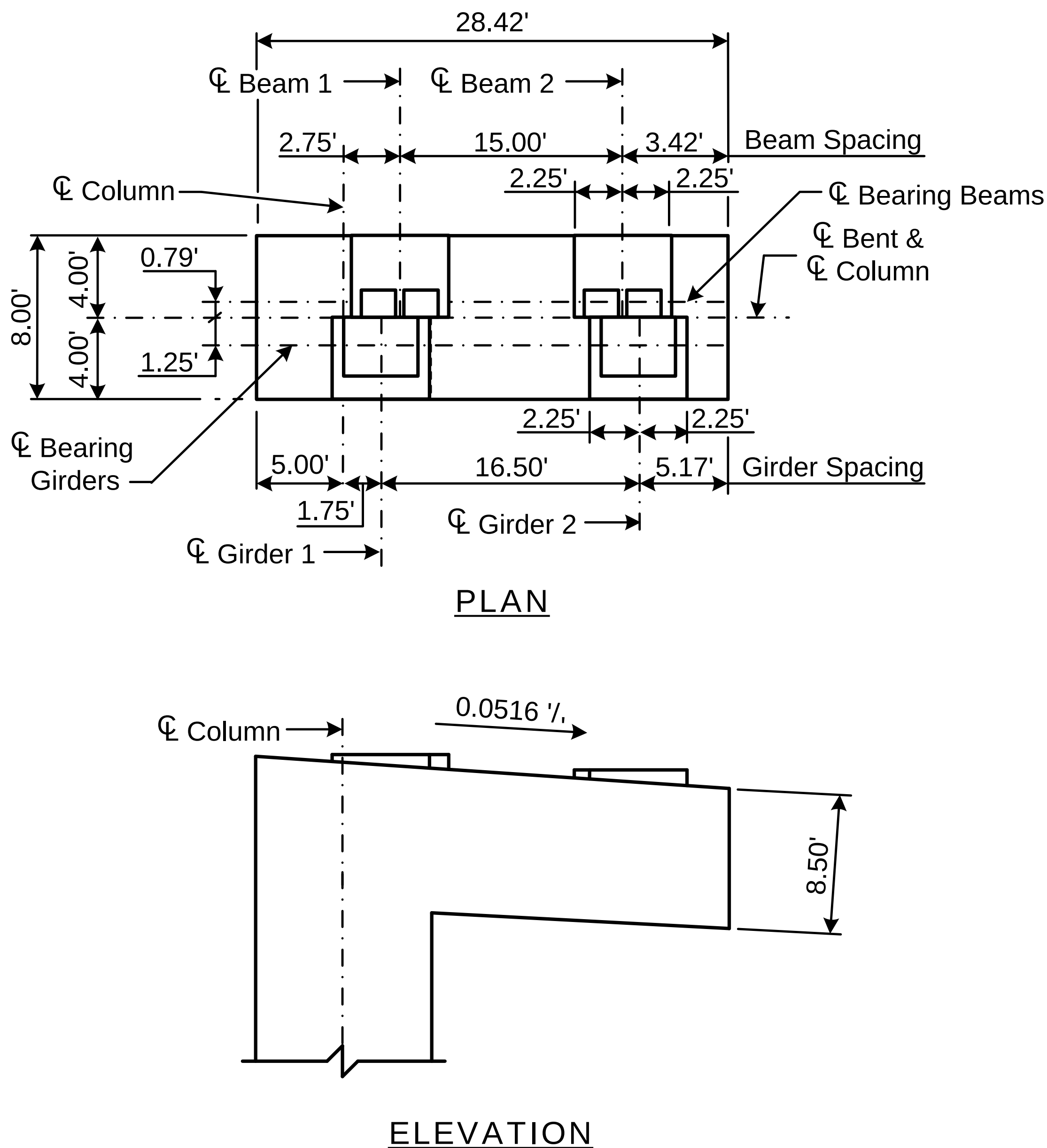


Figure 2-2: Plan and Elevation Views of Cantilever Bent Cap (Detailed Geometry)

Define Material Properties:

Material properties for this design example are as follows:

Concrete strength: $f'_c = 6.0\ ksi$

Reinforcement strength: $f_y = 60\ ksi$

Concrete unit weight: $w_c = 150\ pcf$

Since normal weight concrete is being used and since f'_c is less than 15 ksi and f_y is less than 75 ksi, the AASHTO provisions for STM are applicable (*AASHTO LRFD* Article 5.8.2.1).

> *Design Iterations Using STM:*
>
> When using STM, design iterations may be necessary to determine both the concrete strength and bent cap width to provide adequate strength to the critical node. Since the geometry of the strut-and-tie model is dependent on the value of f'_c and the cap width, the geometry of the STM must be updated for every iteration that is performed. This design example presents the development of final strut-and-tie models for the last iteration that was performed for this problem.

Determine the Bearing Areas:

For this design example, each of the bearing pads supporting the prestressed concrete U-beams is 16 inches by 9 inches. The steel girders are supported by pot bearings with masonry plates that rest on the bearing seats. The sizes of the masonry plates for Girder 1 and Girder 2 are 42 by 29.5 inches and 24 by 24 inches, respectively.

Each bearing pad or plate is placed on a bearing seat that allows the applied force to spread over an area of the cap surface that is larger than the pad or plate itself. The longitudinal dimensions (i.e., effective lengths) of the effective areas are measured at the top surface of the bent cap and labeled in Figure 2-3. A plan view of the bearings is presented in Figure 2-4. The transverse dimensions (i.e., effective widths) shown in Figure 2-4 are measured at the centerline of each bearing pad or plate.

AASHTO LRFD Article 5.6.5 specifies a slope of 1 vertical to 2 horizontal for computing the effective areas. However for simplification, a slope of 1 vertical to 1 horizontal was used in this design example. The effective width of the bearing area of Girder 1 has been limited to prevent overlap with the effective bearing area of Beam 1. The dimensions of the bearing areas are summarized in Table 2-1, along with the size of the effective bearing area for each beam or girder.

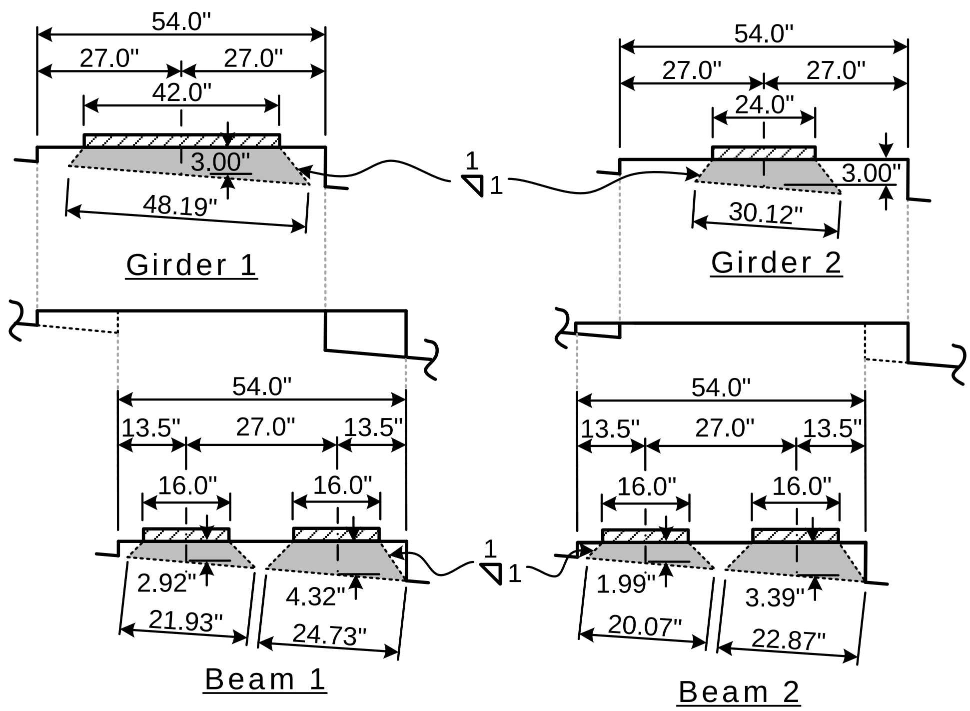


Figure 2-3: Elevation View Showing Effective Bearing Areas Considering Effect of Bearing Seats

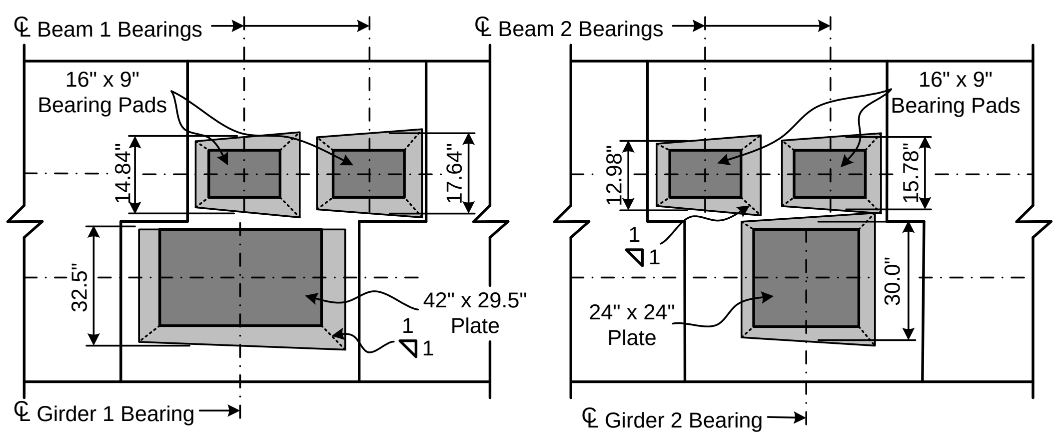


Figure 2-4: Plan View Showing Effective Bearing Areas Considering Effect of Bearing Seats

Table 2-1: Bearing Sizes and Effective Bearing Areas for Each Beam or Girder

Characteristic	Girder 1	Beam 1, Pad 1	Beam 1, Pad 2	Girder 2	Beam 2, Pad 1	Beam 2, Pad 2
Bearing Size	42"x29.5"	16"x9"	16"x9"	24"x24"	16"x9"	16"x9"
Effective Length	48.19"	21.93"	24.73"	30.12"	20.07"	22.87"
Effective Width	32.5"	14.84"	17.64"	30.0"	12.98"	15.78"
Effective Area	1566 in^2	326 in^2	436 in^2	904 in^2	260 in^2	361 in^2

Nodes Directly Below Applied Loads:

A simplification is provided to facilitate definition of the geometry of the nodes that are located directly below the applied superstructure loads. Specifically, the bearing areas are assumed to be square and located concentrically with the longitudinal axis of the bent cap.

Design Step 2 - Determine the Locations of the B- and D-Regions

The entire cantilever bent cap is a D-Region due to the applied superstructure loads (i.e., load discontinuities) and the geometric discontinuity of the frame corner. The behavior of the bent cap is therefore dominated by a nonlinear distribution of strains.

Transition from D-Region to B-Region:

The transition from a D-Region to a B-Region occurs approximately one member depth away from a load or geometric discontinuity (*AASHTO LRFD* Article 5.5.1.2.1).

Considering the bent in this design example, the D-Region/B-Region interface is assumed to be located at a distance of one column width (i.e., 10 feet) from the bottom of the bent cap. The limit of the D-Region is shown in Figure 2-5.

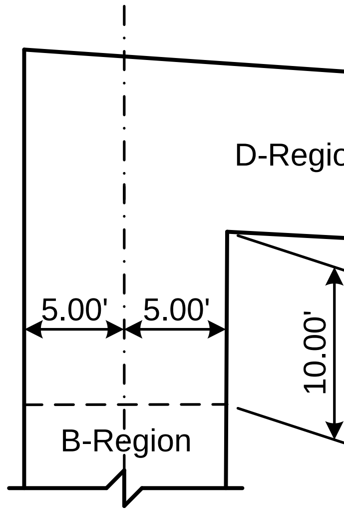


Figure 2-5: Limit of D-Region in Bent Cap

Design Step 3 - Define Load Cases

The factored superstructure loads from the two steel girders and two concrete U-beams are shown in Figure 2-6(a). Loads for both the strength and service limit states are presented. These loads correspond to one particular load case that must be considered during the design process. In this design example, the Girder 1 reaction is significantly greater than the Girder 2 reaction due to various additional loads applied to the fascia girder that are not applied to the interior girder.

The final design of the bent cap must satisfy the design requirements for all governing load cases. The superstructure design loads are assumed to act at the point where the longitudinal centerline of a beam or girder coincides with the transverse centerline of the respective bearing pads.

Resolving Point Loads:

Point loads in close proximity to one another can be resolved together to simplify the load case and facilitate development of a practical strut-and-tie model.

The factored loads on the left and right can be resolved into single loads through superposition, as shown in Figure 2-6(b). The locations of the resolved loads are determined by the calculations presented below. In these calculations, x_1 is the horizontal distance from the centerline of the column to the left resolved load, P_1. Similarly, x_2 is the horizontal distance from the centerline of the column to the right resolved load, P_2, as shown in the plan view of Figure 2-6(b).

$$x_1 = \frac{(404\ kip)(2.75\ ft) + (1396\ kip)(1.75\ ft)}{404\ kip + 1396\ kip} = 1.97\ ft$$

$$x_2 = \frac{(404\ kip)(2.75\ ft + 15.00\ ft) + (367\ kip)(1.75\ ft + 16.50\ ft)}{404\ kip + 367\ kip} = 17.99\ ft$$

The dimensions in the above calculations are illustrated in Figure 2-6(a), and the resolved loads are assumed to act at the longitudinal centerline of the top of the bent cap, as illustrated in Figure 2-6(b).

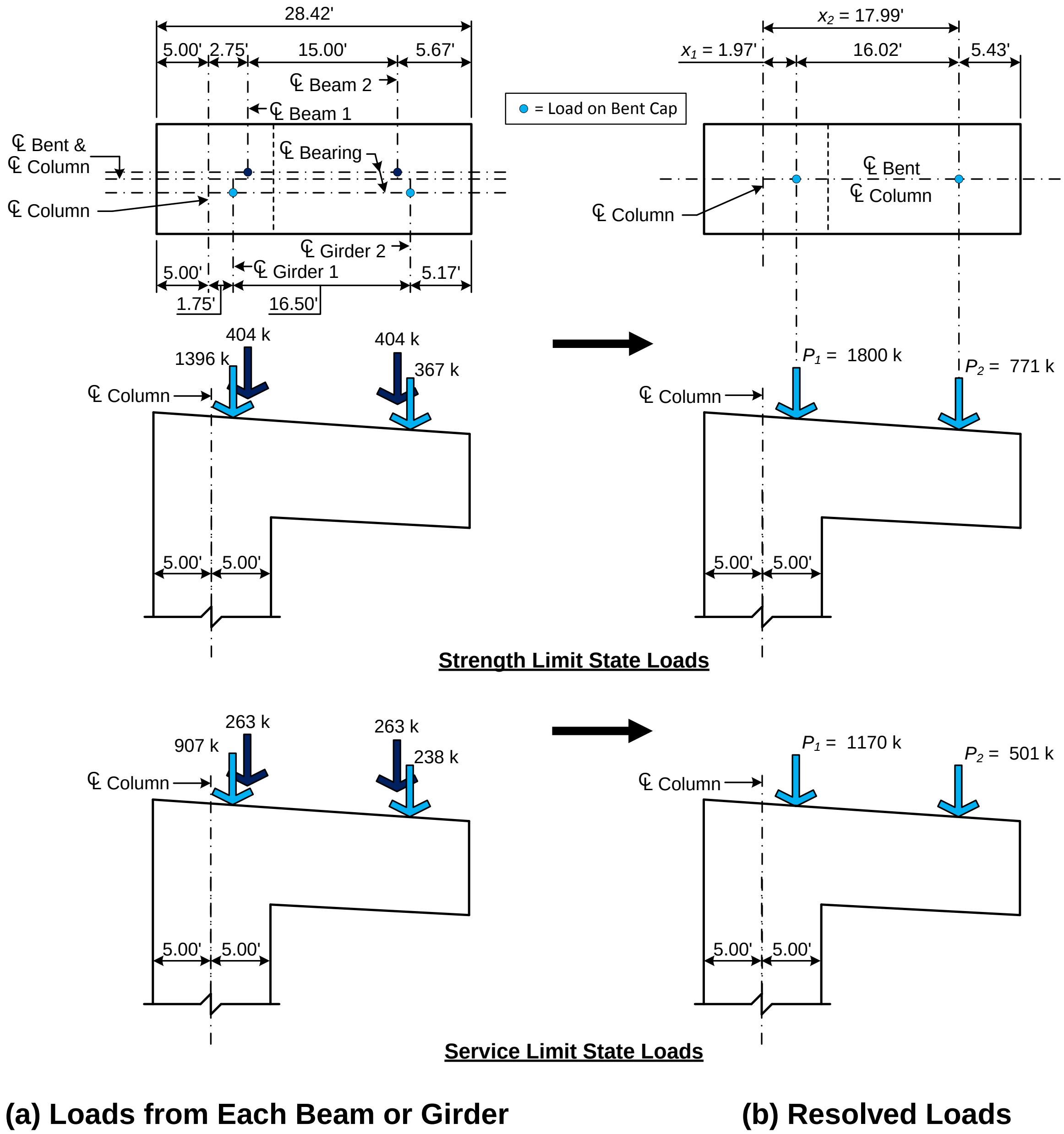


Figure 2-6: Factored Superstructure Loads Acting on Bent Cap

After the design loads from the superstructure acting on the bent cap have been computed, as shown in Figure 2-6(b), the self-weight of the cantilever bent cap must also be considered. The factored self-weight based on tributary volumes is added to each load, as presented in Figure 2-7. As previously defined, the unit weight of the reinforced concrete is 150 pcf. The magnitude of each load acting on the strut-and-tie model, including the self-weight of the bent cap, is computed as follows:

$$P_1 = 1800\ kip + 1.25(164\ kip) = 2005\ kip$$

$$P_2 = 771\ kip + 1.25(124\ kip) = 926\ kip$$

The first value in each calculation is the factored superstructure load, and the second value is the tributary self-weight of the cantilever bent cap factored by 1.25 for the Strength I load combination (*AASHTO LRFD* Tables 3.4.1-1 and 3.4.1-2). These calculations result in the final design loads for the strength limit state acting on the bent cap, as shown in Figure 2-7.

Tributary Volumes:

The tributary volumes in this design example include concrete details (e.g., bearing seats and bent cap end details) not shown for clarity. Taking these details into account, the location of the self-weight dividing line between P_1 and P_2 is approximately 12.2 feet from the right end of the cantilever bent cap, as shown in Figure 2-7. An alternate method is to define the self-weight dividing line as the mid-point between the two applied loads.

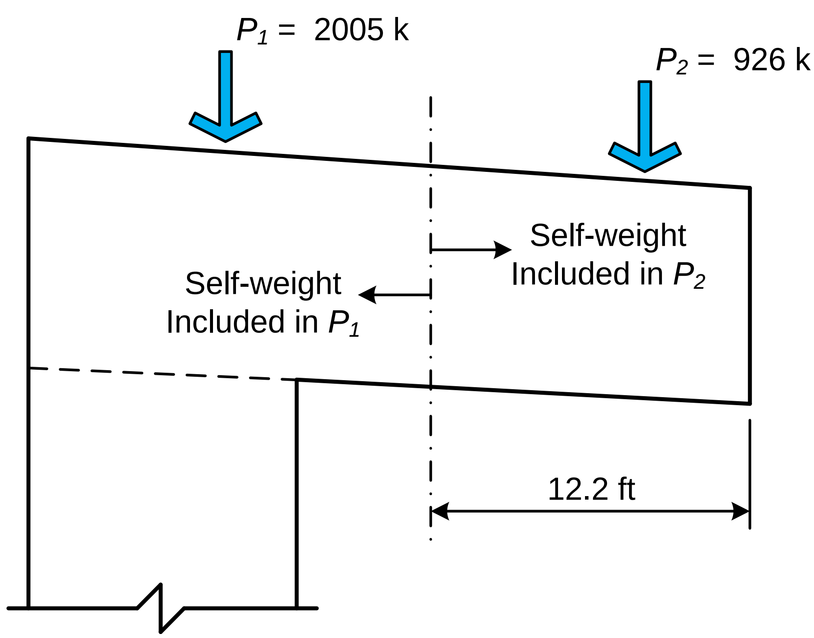


Figure 2-7: Total Factored Loads Based on Superstructure Loads and Bent Cap Self-weight

Referring to Table 2-1 in Design Step 1, the effective bearing area for the load P_1 acting on the bent cap in Figure 2-7 is the combination of the effective bearing areas for Beam 1 and Girder 1, or 2328 in^2, and it is assumed to be a 48.2-inch by 48.2-inch square (i.e., $\sqrt{2328\ in^2} = 48.2\ in$). Similarly, the effective bearing area for the load P_2 acting on the cap is assumed to be a 39.1-inch by 39.1-inch square. Both loads are assumed to act at the center of these effective bearing areas.

Design Step 4 - Analyze Structural Components

Assuming a linear distribution of stress at the interface of the B- and D-Region (based on St. Venant's principle, as per *AASHTO LRFD* Article C5.5.1.2.1), the linear stress distribution is as shown in Figure 2-8 and the extreme fiber stress for the right side of the column is computed as follows:

$$f_{Right} = \frac{P_1 + P_2}{A_{Column}} + \frac{Mc}{I_{Column}}$$

$$M = P_1 x_1 + P_2 x_2 = (2005\ kip)(1.97\ ft) + (926\ kips)(17.99\ ft) = 247{,}303\ kip\text{-}in$$

$$f_{Right} = \frac{2005\ kip + 926\ kip}{(96\ in)(120\ in)} + \frac{(247{,}303\ kip\text{-}in)\ (60\ in)}{13{,}824{,}000\ in^4} = 1328\ psi$$

where:

A_{Column} = cross-sectional area of the column, in.^{2}

I_{Column} = moment of inertia of the column, in.^{4}

M = moment at the centerline of the column due to P_1 and P_2, k-in

c = distance from the extreme fiber to the centerline of the column, in.

It should be noted that at the strength limit state, the concrete stress distribution will not be linear but rather the concrete would be cracked. The designer can therefore treat the boundary section as a cracked section. Doing so, however, would be overly conservative.

For this design example, the linear stress distribution at the interface of the B- and D-Regions is computed based on St. Venant's principle. A primary purpose for calculating the bending stresses in this manner is to define the locations of the struts within the column, and linear stress distribution is used to optimize conservatism. More specifically, if we assumed that the column is at its flexural capacity, and recognizing that the column section has symmetrically distributed reinforcement, the depth of the compression zone would be very small. Although this analysis technique would be acceptable, it would be overly conservative. By justifying a greater compression zone, our solution is sufficiently, but not overly, conservative.

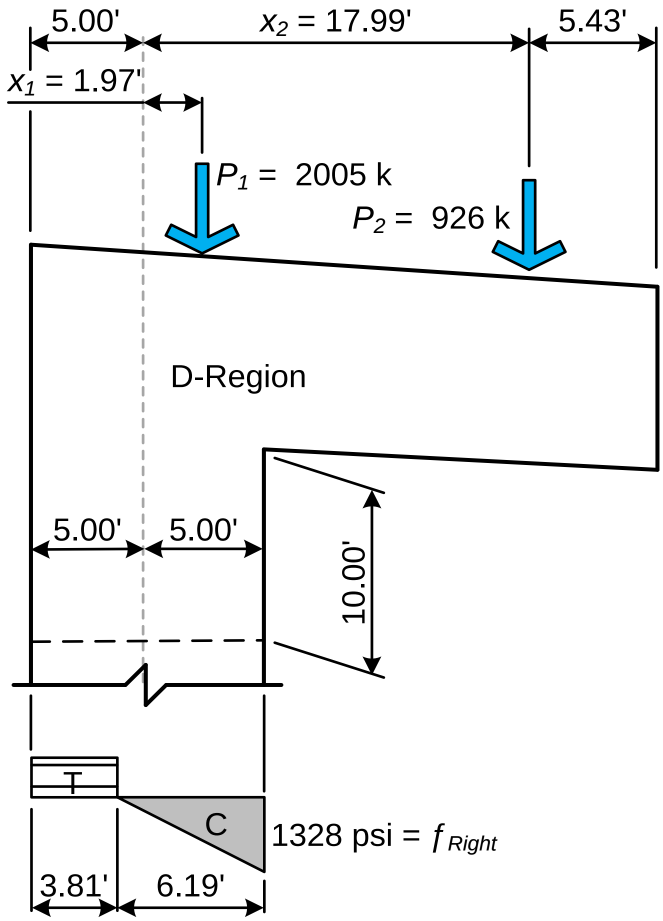


Figure 2-8: Linear Stress Distribution at the Boundary of the B- and D-Regions

Design Step 5 - Size Structural Components Using the Shear Serviceability Check

The likelihood of the formation of diagonal cracks in the cantilevered portion of the bent cap should be considered. To limit diagonal cracking, the service level shear force should be less than the estimated diagonal cracking strength of the member. Using the *AASHTO LRFD* Service I load combination, the service level shear force is computed at the inside face of the column. Based on Figure 2-6, the service limit state value of P_2 is 501 kips, and the self-weight of the cantilever portion of the bent cap is 188 kips. Therefore, the total service level shear force at the inside face of the column is 689 kips.

A value for *d* must be assumed at this point. It is assumed that a cover of 2 inches is provided, No. 6 stirrups are used, No. 11 longitudinal rebar is used in two layers, and a clear space of 2 inches is provided between layers. Thus the value for *d* is found as:

$$d = 102 - 2 - 0.75 - 1.41 - 1 = 96.8\ in$$

The distance *a* is known as the *shear span*. In this design example, the shear span is the distance from the applied load, P_2, to the right face of the column. Therefore, as illustrated in Figure 2-8, *a* is equal to 12.99 feet or 155.9 inches.

As specified in *AASHTO LRFD* Equation C5.8.2.2-1, the estimated resistance at which diagonal cracks begin to form, V_{cr}, for the cantilever portion of the bent cap is computed as follows:

$$V_{cr} = \left[0.2 - 0.1\left(\frac{a}{d}\right)\right]\sqrt{f'_c}b_w d = [0.2 - 0.1\left(\frac{155.9\ in}{96.8\ in}\right)]\sqrt{6\ ksi}(96\ in)(96.8\ in) = 887\ kip$$

where:

a = shear span = 155.9 in. (as previously described and computed)

d = effective depth of the member = 96.8 in. (as previously described and computed)

In addition to the equation presented above, V_{cr} must not be greater than $0.158\sqrt{f'_c}b_w d$ nor less than $0.0632\sqrt{f'_c}b_w d$ (*AASHTO LRFD* Article C5.8.2.2).

$$0.158\sqrt{f'_c}b_w d = 0.158\sqrt{6\ ksi}(96\ in)(96.8\ in) = 3596\ kip > 887\ kip$$

$$0.0632\sqrt{f'_c}b_w d = 0.0632\sqrt{6\ ksi}(96\ in)(96.8\ in) = 1439\ kip > 887\ kip$$

Therefore, $V_{cr} = 1439\ kip > 689\ kip \quad \boldsymbol{OK}$

The estimated diagonal cracking resistance is considerably greater than the service level shear force. Therefore, diagonal cracks are not expected to form under the service loads considered in this design example. If this check did not produce a favorable result, the designer could resize the cap and/or increase the compressive strength of concrete to satisfy this design check.

The equation for V_{cr} presented in *AASHTO LRFD* Article C5.8.2.2 is based on shear resistance and does not consider torsional effects. If significant torsion is present, the V_{cr} expression can be modified by taking torsion into account (not currently defined in *AASHTO LRFD*). For this design example, since torsion is not significant, its effects need not be considered.

Design Step 6 - Develop a Strut-and-Tie Model

For this design example, two STMs are used to model the flow of forces within the cantilevered portion of the bent. The first model, a direct strut model shown in Figure 2-9, features one truss panel in the cantilevered portion and models a direct flow of forces to the column. As defined in *AASHTO LRFD* Article 5.8.2.2, the angle between the axes of a strut and tie should be limited to angles greater than 25 degrees. As presented in Figure 2-9, the angle between the strut and tie for the direct strut model is 28.3 degrees. Therefore, the direct strut model for this design example satisfies the AASHTO requirements. In addition, for this design example, the direct strut model is a more efficient and correct model, and it better represents the actual flow of forces within the cantilever bent cap.

However, simply as a learning exercise, a two panel model is presented in Figure 2-10. This second model features two truss panels with an intermediate vertical tie and was developed to investigate the requirements of the vertical tie within the cantilever when using a two panel model. All other characteristics of the STM geometry are the same for both models. The description of the STM development presented in this section applies primarily to the first model, shown in Figure 2-9, unless otherwise noted. As specified in *AASHTO LRFD* Article C5.8.2.2, the designer should minimize the number of vertical ties between a load and a support using the least number of truss panels possible while still satisfying the 25 degree minimum. Since the direct strut model satisfies the 25 degree minimum requirement, the two panel model is not necessary and is not recommended by AASHTO. In addition, for this design example, the two panel model is a less efficient and less correct model, and it does not accurately represent the actual flow of forces within the cantilever bent cap. It is included in this design example solely as a learning exercise.

As previously explained, the geometry of the STM is dependent on the value of f'_c and the cap geometry, and the STM must correspond to the applied loads and chosen geometry. The following explanation applies to the development of the final STMs for the last iteration that was performed.

In Figures 2-9 and 2-10, the width of the trapezoid defining the location of Strut *E-E'* is determined by setting the trapezoidal stress volume equal to the value of P_2, as follows:

$$\left(\frac{1}{2}\right)(1328\,psi)[1+\left(\frac{6.19\,ft-x}{6.19\,ft}\right)]x(96\,in)\left(\frac{12\,in}{ft}\right)\left(\frac{1\,kip}{1000\,lb}\right)=926\,kip$$

$$x = 0.64\,ft$$

The locations of Struts *E-E'* and *D-D'* can then be determined using basic centroid equations found in geometry books. The location of Strut *E-E'*, determined by computing the centroid of the trapezoidal stress volume defined above, is 0.31 feet from the right side of the column. The location of Strut *D-D'*, determined by computing the centroid of the remaining triangular compressive stress volume, is 2.17 feet from Strut *E-E’*. These values are illustrated in Figures 2-9 and 2-10.

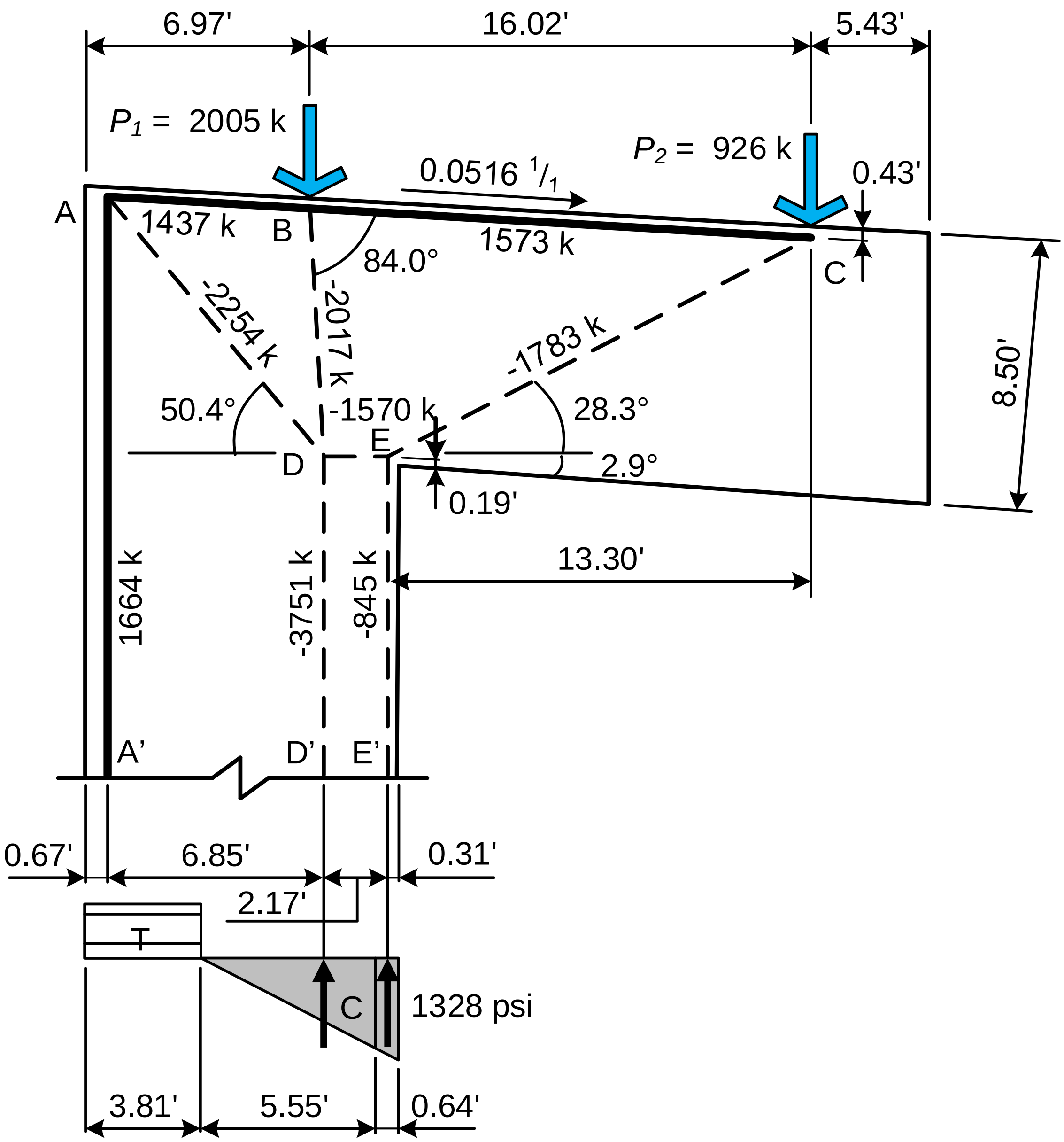


Figure 2-9: Strut-and-Tie Model Featuring One Truss Panel for the Cantilever Bent Cap

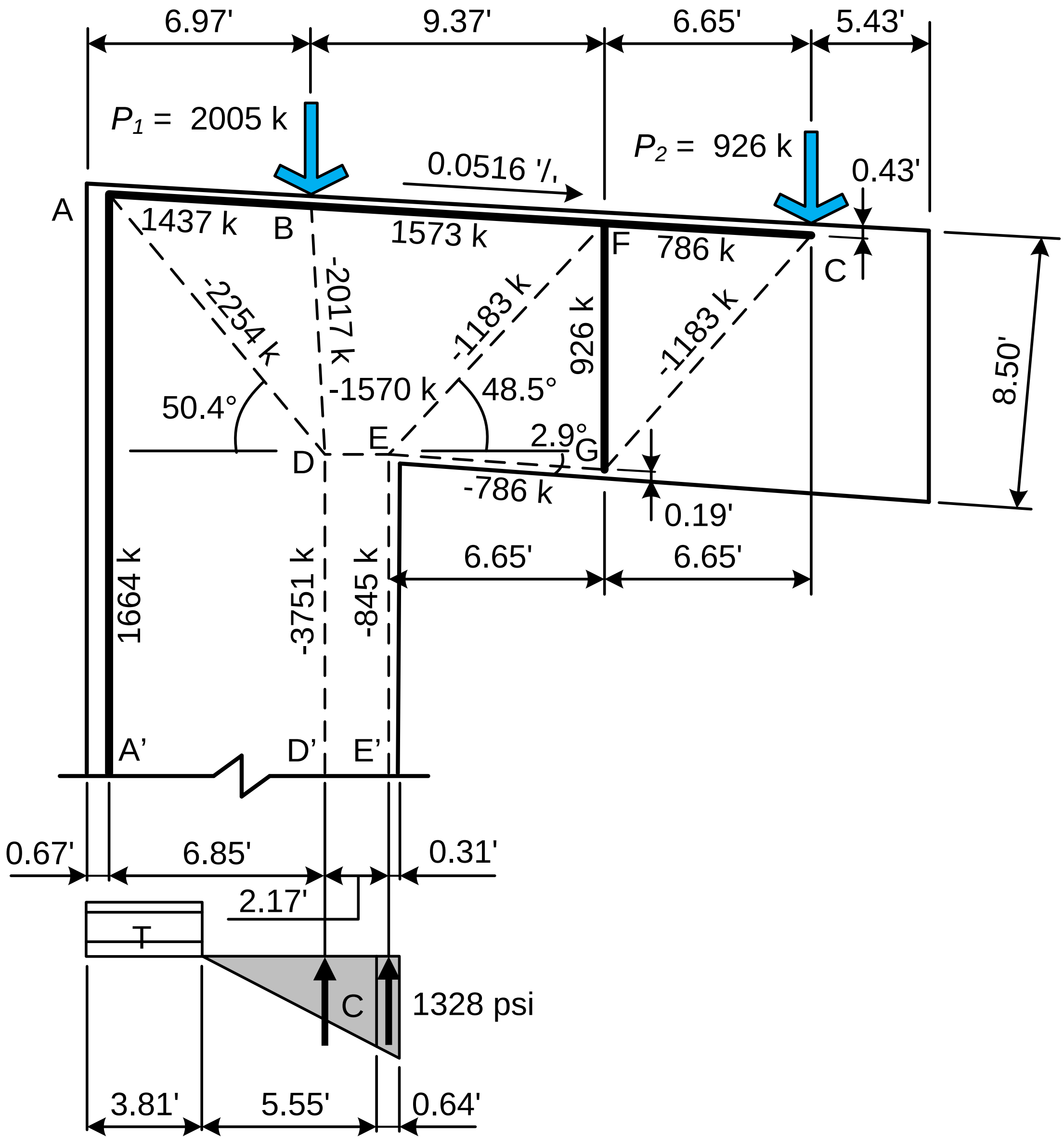


Figure 2-10: Strut-and-Tie Model Featuring Two Truss Panels for the Cantilever Bent Cap

Placement of Struts:

The first step in developing the STMs is to determine the locations of the vertical struts within the column, identified as Struts *DD'* and *EE'* in Figures 2-9 and 2-10. As previously described, the struts are located to correspond with the resultants of the compressive portion of the linear stress diagram at the boundary of the D-Region.

If a single strut is used to model the forces within the column, as illustrated in Figure 2-11(a), it should be positioned to correspond with the resultant of the compressive portion of the linear stress diagram. For this design example, if only P_2 acted on the bent cap, one vertical strut within the column would be sufficient. However, since there are two applied loads, a second vertical strut is needed to model the direct transfer of load P_1 into the column, as shown in Figure 2-11(b).

> *Placement of Struts:*
>
> Correct strut-and-tie models are developed by making a series of rational assumptions. Further, there is generally not one unique solution (or strut-and-tie model) for each case. There is much flexibility associated with STM design. For this design example, the use of a single strut coupled with the cracked column section at flexural ultimate gives a very conservative size to the flexural compression zone. Therefore, the designer has two options: (1) use an oversized column to satisfy a perceived problem triggered by one STM, or (2) look for other load paths (a different STM) that demonstrates the fact that the actual column size is in fact acceptable. For this design example, the second option is applied, and two vertical struts in the column are used to model the direct transfer of the two applied loads.

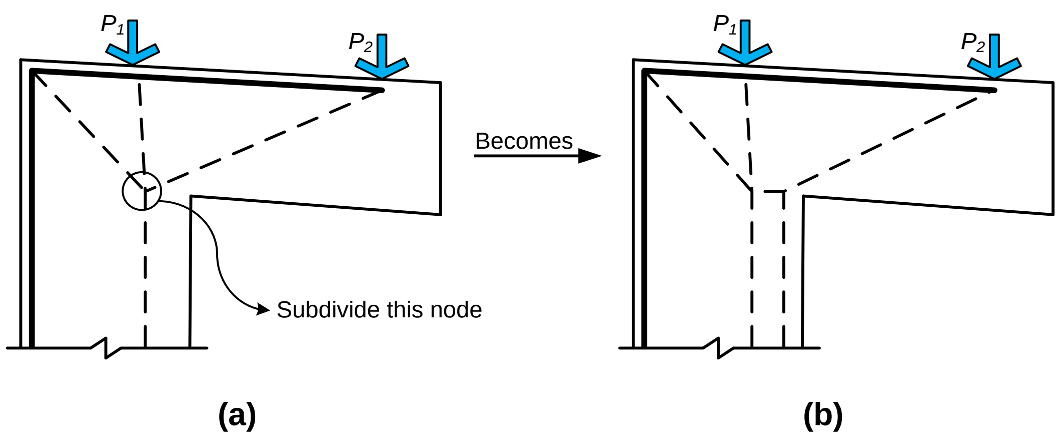


Figure 2-11: Modeling Compressive Forces within the Column: (a) Using a Single Strut and (b) Using Two Struts

In order to position the two vertical struts within the column, the compressive portion of the stress diagram is subdivided into two parts, a trapezoidal shape and a triangular shape, as shown in Figures 2-9 and 2-10. The geometry of each subdivision is determined by setting its resultant force equal to the corresponding force within the structure. The resultant of the trapezoidal shape at the right is generally equal to the magnitude of P_2, and the resultant of the triangular shape is generally equal to P_1 plus the resultant of the tensile portion of the stress diagram. For this design example, however, the resulting forces in the vertical struts within the column (Struts *DD'* and *EE'*) do not equal the resultants of the stress diagram subdivisions that were previously determined. This is to be expected since Tie *AA'* within the column must coincide with the column reinforcement and therefore does not coincide with the resultant of the tensile portion of the stress diagram. The slight angle of Ties *AB* and *BC* also contributes to the difference in forces. The combined effect of the forces in Strut *DD'*, Strut *EE'*, and Tie *AA'*, however, is equivalent to the axial force and moment within the column at the D-Region/B-Region interface. The strut-and-tie models, therefore, satisfy the STM requirements for design.

Placement of Ties:

The next step in developing the STMs is to determine the placement of Ties *AB*, *BC*, and *AA'* in Figure 2-9.

The locations of the ties must correspond with the centroids of the longitudinal tension steel that will be provided within the structure (*AASHTO LRFD* Article C5.8.2.2).

Design iterations are generally needed to achieve this level of accuracy. When using the STM procedure, the designer should compare the final reinforcement details (i.e., the centroids of the longitudinal reinforcement) with the locations of the longitudinal ties of the STM to decide whether another iteration would affect the final design.

As previously explained, it is assumed that a cover of 2 inches is provided, No. 6 stirrups are used, No. 11 longitudinal rebar is used in two layers, and a clear space of 2 inches is provided between layers. The distance from the top surface of the bent cap to the centroid of the reinforcement along the top of the bent cap is therefore computed as follows:

$$2 + 0.75 + 1.41 + 1 = 5.16 \text{ in}$$

The centroid of the main tension steel within the column is assumed to be located 8.0 inches from the left face of the column. Considering the final reinforcement layout presented in Figures 2-19 and 2-20 following the STM design, the locations of Ties *AB* and *BC* described above correspond with the centroids of the main longitudinal reinforcement within the bent cap.

Placement of the Corner Node (Node* E*):

Before the remaining members of the STM are positioned, the location of Node *E* should be determined. The horizontal position of Node *E* is defined by the location of the vertical strut near the right face of the column (Strut *EE'*). Only the vertical position of the node, therefore, needs to be determined. In contrast to the placement of the column struts, a linear distribution of stress cannot be used to position the node since no D-Region/B-Region interface exists within the cap (i.e., the entire cap is a D-Region). The vertical position of Node *E* is therefore defined by optimizing the height of the STM (i.e., the moment arm, *jd*, of the bent cap) to achieve efficient use of the bent cap depth. Node *E* is placed so that the factored force acting on the back face will be approximately equal to its design strength. In other words, the moment arm, *jd*, is as large as possible while still ensuring that the back face of Node *E* has adequate strength. The calculation necessary to determine the vertical location of Node *E* is shown below and is illustrated in Figure 2-12. The moment at the right face of the column due to load P_2 (neglecting the slight angle of the bent cap) is set equal to the factored resistance of the back face of Node *E* times the moment arm, *jd*.

$$P_2(12.99\ ft) = \overbrace{\phi v f'_c b_w a}^{\text{Back face design strength}} \overbrace{\left(d - \frac{a}{2}\right)}^{jd}$$

$$926\ kip(12.99\ ft) = (0.7)(0.85)(6.0\ ksi)(96\ in)a\left(96.8\ in - \frac{a}{2}\right)$$

$$a = 4.46\ in$$

The resistance factor, ϕ, in the calculation is the *AASHTO LRFD* factor of 0.7 for compression in strut-and-tie models (*AASHTO LRFD* Article 5.5.4.2). The concrete efficiency factor, *v*, is taken as the factor for the back face of Node *E* (0.85 for a CCC node, as presented in *AASHTO LRFD* Table 5.8.2.5.3a-1). The term left of the equal sign is the moment at the right face of the column. The vertical location of Node *E* is taken as 2.23 inches from the bottom face of the bent, a distance equal to *a*/2. The exact location of Node *E* is clearly shown in Figure 2-14 (in the section presenting the nodal strength checks for Node *E*).

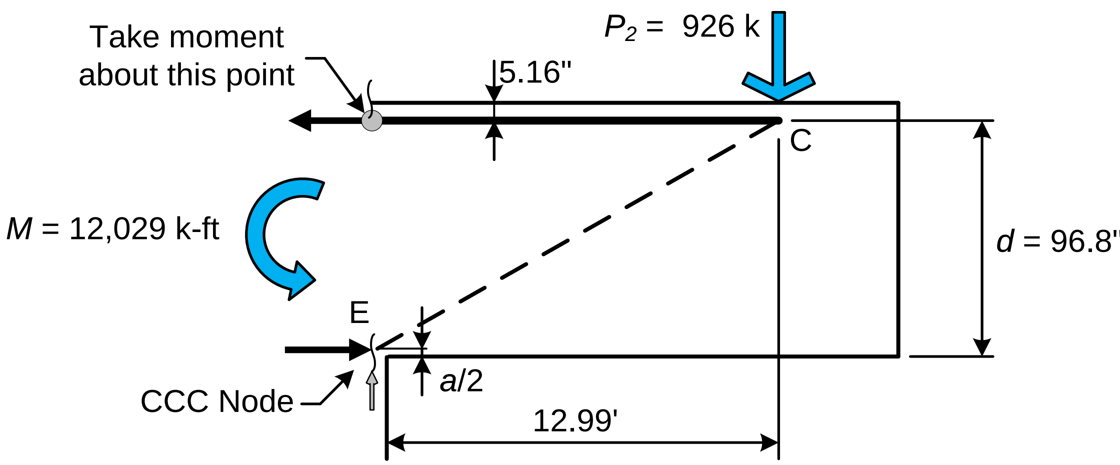


Figure 2-12: Determining the Vertical Position of Node *E*

Placement of the Remaining Nodes:

The remaining nodes within the strut-and-tie model shown in Figure 2-9 can now be positioned. Node *D* is located vertically to align with Node *E*, and it is located horizontally to align with Node *D'*. Strut *DE* connects the two nodes. Nodes *B* and *C* are located vertically below the applied superstructure loads. Struts *AD*, *BD*, and *CE* are then added to model the elastic flow of forces within the bent cap. These struts connect the nodes that have already been defined.

Similarly, the remaining nodes within the strut-and-tie model shown in Figure 2-10 can also be positioned. The vertical Tie *FG* is located midway between Strut *EE'* and Node *C*. Strut *EG* is parallel to the bottom face of the bent cap at a distance of 2.23 inches from the face.

Compute the Member Forces:

After the geometry of the STMs has been determined as described above, the member forces of the struts and ties are computed by enforcing equilibrium.

> *Computing the Member Forces:*
>
> Since both models are statically determinate systems, all member forces can be calculated by satisfying equilibrium at the joints of the truss. This can be accomplished using the method of joints. Given the small number of joints, the forces can easily be determined using hand calculations.
>
> It should be noted that these are not "real trusses" in that they do not satisfy the requirements of stability. If this truss was modeled in a computer program, it would not generate results due to the presence of incomplete panels and triangles. Dummy members would need to be added for the computer to generate results. For this STM example, this truss is superimposed on a rigid element. While it provides a convenient way to visualize the flow of forces, it does not necessarily need to be stable in and of itself.

Design Step 7 - Proportion Ties

The only significant difference between the two models presented in Figures 2-9 and 2-10 is the additional vertical Tie *FG* in Figure 2-10. Since a vertical tie is not provided within the cantilevered portion of the bent cap in Figure 2-9, the STM in Figure 2-10 was developed as a learning exercise to determine the amount of stirrups required to resist the force in Tie *FG*. However, since we previously determined that the direct strut model in Figure 2-9 satisfies AASHTO requirements, the design results from that model will be used for final design.

Design Requirement for Tie **FG** *Using Two Truss Panels:*

The nodes at the ends of the vertical tie, Nodes *F* and *G*, are both smeared nodes. Although the strut-and-tie truss model implies that struts and ties occupy a specific location, in reality the stresses spread out over portions of the member as shown in Figure 2-13. *AASHTO LRFD* suggests that an "available length" be determined over which the stirrups can be distributed (*AASHTO LRFD* Figure C5.8.2.2-2). By spreading the vertical steel out, the individual bars better resist the overall distribution of forces as the concentrated load from the reaction and applied load fan out across the cantilever bent cap depth and length. The 25-degree angle in Figure 2-13 comes from the fact that compression is assumed to spread at 2:1 (vertical to horizontal) within the member. This corresponds to an angle of 26.6 degrees, which is rounded to 25 degrees in *AASHTO LRFD* Article 5.8.2.2.

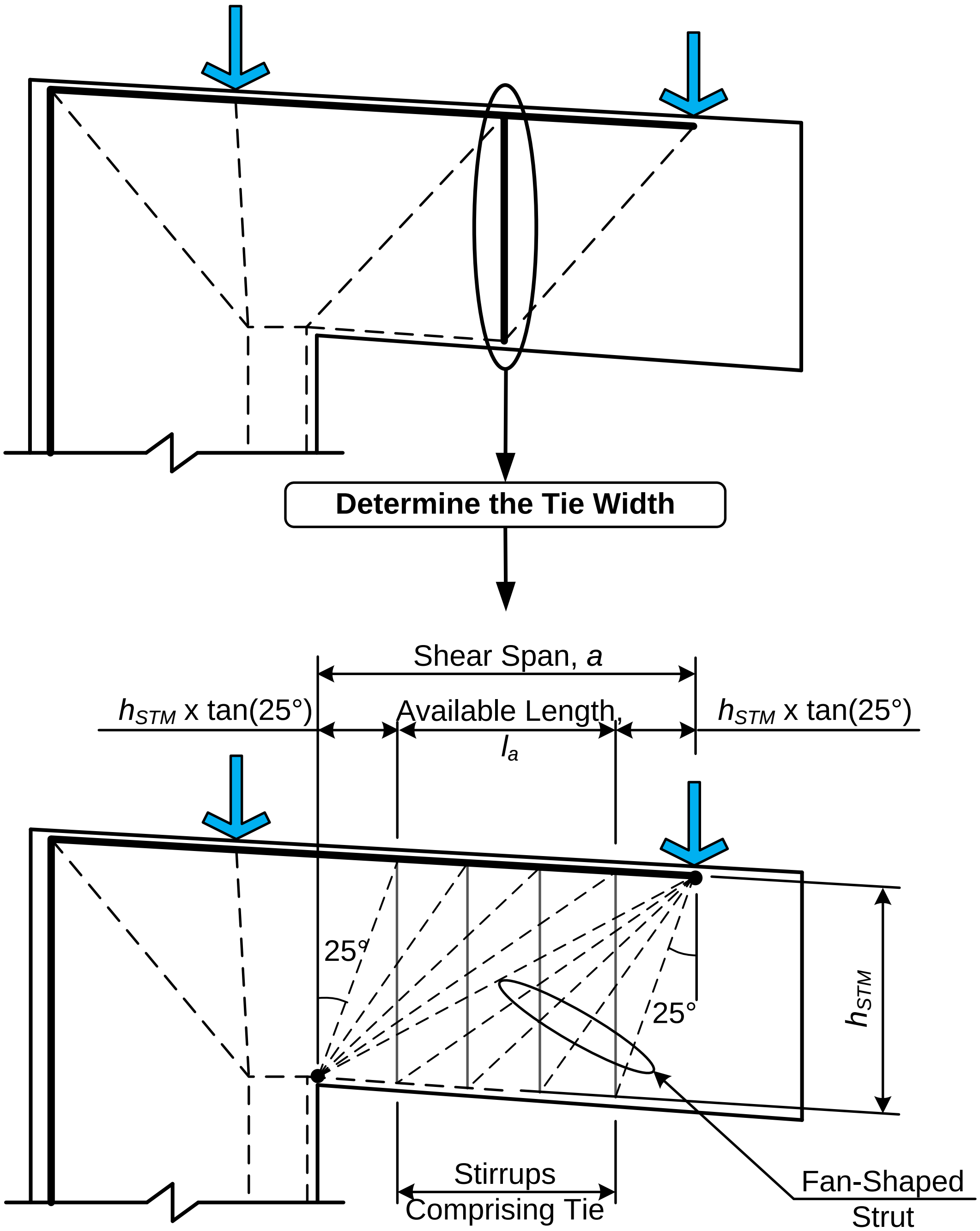


Figure 2-13: Fan-Shaped Struts Engaging Reinforcement, Forming a Tie

The available length over which the reinforcement comprising Tie *FG* can be distributed is therefore computed as follows:

$$l_a = (12.99\ ft)(12\frac{in}{ft}) - 2(94.6\ in)(tan25°) = 67.7\ in$$

where 12.99 feet is the shear span, *a*, as previously described, and 94.6 inches is the vertical distance between Nodes *F* and *G*, or the length of Tie *FG*. The value of l_a is slightly conservative because the cross slope of the bent cap is ignored in its calculation.

The resistance factor, ϕ, for tension in strut-and-tie models for reinforced concrete is 0.9 (*AASHTO LRFD* Article 5.5.4.2). The strength of the ties must satisfy the following equation (*AASHTO LRFD* Equations 5.8.2.3-1 and 5.8.2.4.1-1):

$$P_r = \phi \cdot P_n = \phi \cdot f_y \cdot A_{st}$$

Distributing four-legged #6 stirrups over the available length, the required spacing necessary to carry the force in Tie *FG* is determined as follows:

Factored load: $P_u = 926\ kip$

Tie capacity: $\phi \cdot f_y \cdot A_{st} = P_u$

$(0.9)(60\ ksi)A_{st} = 926\ kip$

$A_{st} = 17.2\ in^2$

The number of #6 stirrups (with 4 legs each) required and its corresponding stirrup spacing are then computed as follows:

$$17.2\ in^2 / (4)(0.44\ in^2) = 9.8 \text{ stirrups}$$

$$l_a = 67.7\ in$$

$$s = 67.7\ in / 9.8\ stirrups = 6.9\ in\ spacing$$

Therefore, the spacing of four-legged #6 stirrups should be no greater than 6.9 inches to satisfy the requirements for Tie *FG* for the STM with two truss panels, as depicted in Figure 2-10.

Design Requirement for Crack Control Reinforcement:

The stirrup requirements previously computed for the STM with two truss panels will now be compared to the minimum crack control reinforcement requirement. The crack control reinforcement must satisfy the following two equations (*AASHTO LRFD* Equations 5.8.2.6-1 and 5.8.2.6-2):

$$A_v / b_w s_v \geq 0.003$$

$$A_h / b_w s_h \geq 0.003$$

where:

A_v = total area of vertical crack control reinforcement within spacing s_v, in.^{2}

A_h = total area of horizontal crack control reinforcement within spacing s_h, in.^{2}

s_v = spacing of vertical crack control reinforcement, in.

s_h = spacing of horizontal crack control reinforcement, in.

b_w = width of member web, in.

Using four-legged #6 stirrups, the required spacing of the vertical crack control reinforcement is computed as follows:

$$A_{v1} = 0.003 b_w s_{v1}$$

$$4(0.44\ in^2) = 0.003(96\ in) s_{v1}$$

$$s_{v1} = 6.1\ in$$

It should be noted that the stirrup spacing necessary for Tie *FG* as previously computed is greater than the stirrup spacing necessary for crack control reinforcement. Therefore, if the two panel model in Figure 2-10 were being used to design the cantilever bent cap, the crack control reinforcement would be sufficient to resist the force in Tie *FG*. However, since the direct strut model in Figure 2-9 is acceptable for this design example, this comparison is simply a learning exercise.

The vertical crack control reinforcement detailed above (i.e., four-legged #6 stirrups) will be used throughout the bent cap with the single exception of the region directly above the column. In the region above the column, two-legged #8 stirrups will be used to alleviate congestion and enhance constructability. The required spacing of the vertical crack control reinforcement above the column is computed as follows:

$$A_{v2} = 0.003 b_w s_{v2}$$

$$2(0.79\ in^2) = 0.003(96\ in) s_{v2}$$

$$s_{v2} = 5.5\ in$$

Finally, the required spacing of #8 bars provided as horizontal crack control reinforcement is computed as follows:

$$A_h = 0.003 b_w s_h$$

$$2(0.79\ in^2) = 0.003(96\ in) s_h$$

$$s_h = 5.5\ in$$

The required crack control reinforcement is used along the entire length of the bent cap.

Summary:

Based on the above design computations, the following reinforcement will be used in the bent cap:

- Use 4 legs of #6 stirrups with spacing less than 6.1 inches within the cantilevered portion of the bent cap
- Use 2 legs of #8 stirrups with spacing less than 5.5 inches above the column
- Use #8 bars with spacing less than 5.5 inches as horizontal crack control reinforcement

Final reinforcement details are provided in Figures 2-18, 2-19, and 2-20.

Proportion Longitudinal Ties* AB *and* BC*:

Since the forces in Ties *AA'*, *AB*, and *BC* are all similar, a constant amount of reinforcement will be provided along the top of the bent cap and then down the tension face of the column.

For the longitudinal reinforcement along the top of the bent cap, the force in Tie *BC* controls. Two layers of #11 bars will be provided. The reinforcement is proportioned as follows:

Factored load: $P_u = 1573\ kip$

Tie capacity: $\phi \cdot f_y \cdot A_{st} = P_u$

$$(0.9)(60\ ksi)A_{st} = 1573\ kip$$
$$A_{st} = 29.1\ in^2$$

The number of required #11 bars is then computed as follows:

$$29.1\ in^2 / 1.56\ in^2 = 18.7\ bars$$

Therefore, use 20 - #11 bars in two layers for the longitudinal reinforcement along the top of the bent cap.

Proportion Column Tie* AA'*:

Similarly, for the reinforcement in the column comprising Tie *AA'*, two layers of #11 bars will be provided as the main tension steel. The reinforcement is proportioned as follows:

Factored load: $P_u = 1664\ kip$

Tie capacity: $\phi \cdot f_y \cdot A_{st} = P_u$

$$(0.9)(60\ ksi)A_{st} = 1664\ kip$$
$$A_{st} = 30.8\ in^2$$

The number of required #11 bars is then computed as follows:

$$30.8\ in^2 / 1.56\ in^2 = 19.8\ bars$$

Therefore, use 20 - #11 bars in two layers for the tension reinforcement in the column.

Requirements for Final Reinforcement Details:

The calculated amount of main column tension reinforcement is only satisfactory for the load case under consideration and the STM analysis that was performed. The final reinforcement details for the column are dependent on the complete design that considers all governing load cases and applicable articles in *AASHTO LRFD*.

Design Step 8 - Perform Nodal Strength Checks

The strength of each node of the STM is now checked to ensure that it is sufficient to resist the applied forces. The limiting compressive stress at the node face, f_{cu}, is computed as follows (*AASHTO LRFD* Equation 5.8.2.5.3a-1):

$$f_{cu} = m \cdot v \cdot f'_c$$

where:

m = confinement modification factor (described later in this design example)

v = concrete efficiency factor (*AASHTO LRFD* Table 5.8.2.5.3a-1)

f'_c = compressive strength of concrete, ksi

Node* E *(CCC):

Due to the limited geometry of Node *E* and the high forces it resists, it is identified as the most critical node of the STM. The geometry of Node *E* is detailed in Figure 2-14. Referring back to Figure 2-9, the lateral spread of Strut *EE'* at Node *E* will be limited by the right face of the column. The bottom bearing face of Node *E* (and the width of Strut *EE'*) is therefore taken as twice the distance from the centroid of Strut *EE'* to the right face of the column, or 2(3.76 inches) = 7.5 inches.

The length of the back face, or vertical face, of Node *E* is double the vertical distance from the center of Node *E* (i.e., the point where the centroids of the struts meet) to its bottom bearing face. This length can be calculated as follows:

$$a = 2\left[\frac{2.23\ in}{cos2.9°} + (3.76\ in)tan2.9°\right] = 4.9\ in$$

where 2.9° is the angle between the longitudinal axis of the cap and the horizontal (i.e., the cross slope of the cap). The other dimensions are shown in Figures 2-9 and 2-14.

As illustrated in Figure 2-14, the length of the strut-to-node interface, w_s, where Strut *CE* enters Node *E* is computed as follows:

$$w_s = l_b \sin\theta + a\cos\theta = (7.5\ in)\sin 28.3° + (4.9\ in)\cos 28.3° = 3.6\ in + 4.3\ in = 7.9\ in$$

The use of a computer-aided design program can facilitate determination of the geometry of such a node. A designer can calculate the nodal geometry, as has been done in this design example, or in more complicated cases, the designer can draw it in a CAD platform and let the CAD program perform the descriptive geometry work.

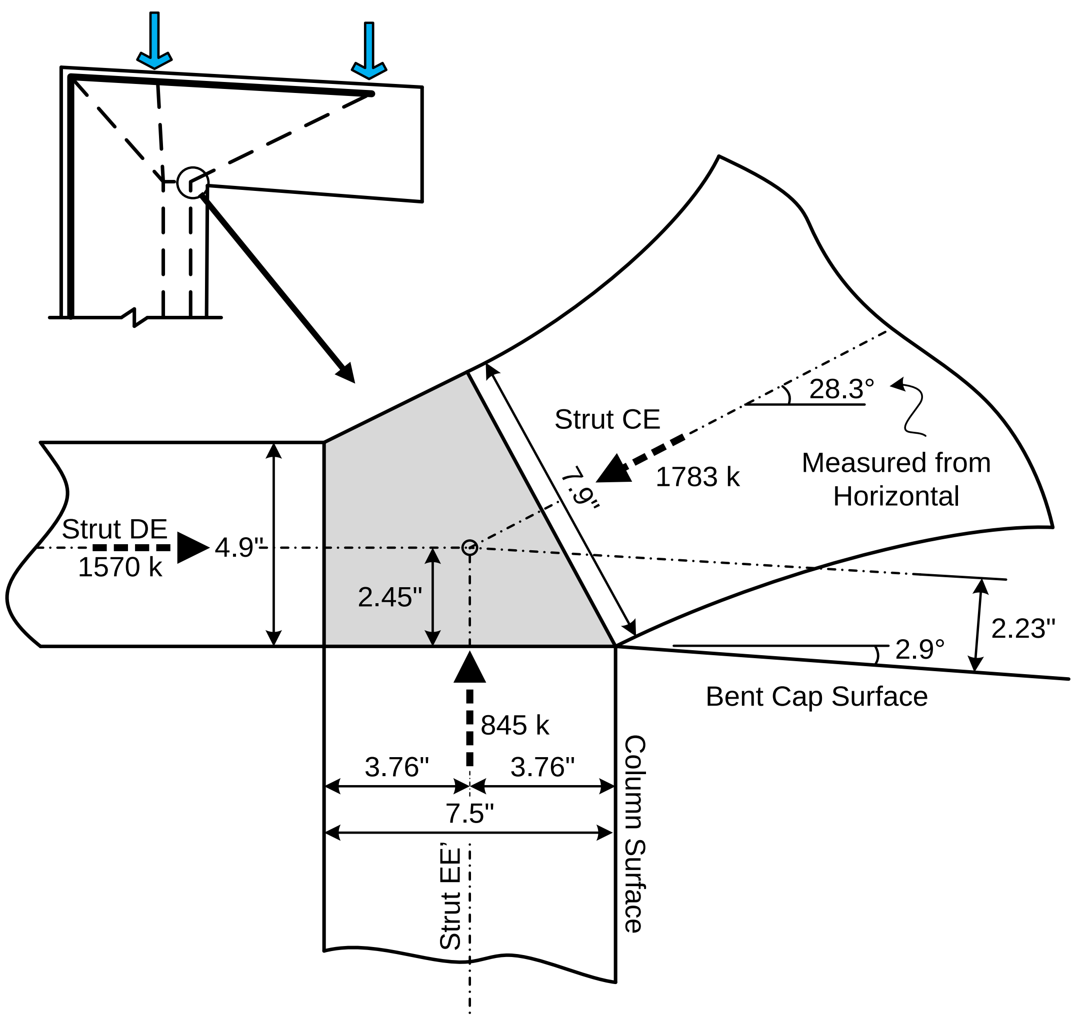


Figure 2-14: Geometry and Forces at Node *E*

Node *E* is a CCC node with concrete efficiency factors of 0.85 for the bearing and back faces and 0.55 for the strut-to-node interface, as specified in *AASHTO LRFD* Table 5.8.2.5.3a-1 (see calculation below). The confinement modification factor, *m*, is 1 since the column and the bent cap have the same width. The faces of Node *E* are checked as follows:

Confinement modification factor: $m = 1$

Cap-to-column bearing face:

Factored load: $P_u = 845\ kip$

Concrete efficiency factor: $\nu = 0.85$

Concrete capacity:

$$f_{cu} = m \cdot \nu \cdot f'_c = (1)(0.85)(6.0\ ksi) = 5.1\ ksi$$

$$\Phi \cdot P_n = (0.7)(5.1\ ksi)(7.5\ in)(96\ in) = 2570\ kip > 845\ kip \quad \boldsymbol{OK}$$

Back face:

Factored load: $P_u = 1570\ kip$

Concrete efficiency factor: $\nu = 0.85$

Concrete capacity:

$$f_{cu} = m \cdot \nu \cdot f'_c = (1)(0.85)(6.0\ ksi) = 5.1\ ksi$$

$$\Phi \cdot P_n = (0.7)(5.1\ ksi)(4.9\ in)(96\ in) = 1679\ kip > 1570\ kip \quad \boldsymbol{OK}$$

Strut-to-node interface:

Factored load: $P_u = 1783\ kip$

Concrete efficiency factor: $\nu = 0.85 - {}^{6.0\ ksi}/_{20\ ksi} = 0.55$

∴ Use $\nu = 0.55$

Concrete capacity:

$$f_{cu} = m \cdot \nu \cdot f'_c = (1)(0.55)(6.0\ ksi) = 3.3\ ksi$$

$$\Phi \cdot P_n = (0.7)(3.3\ ksi)(7.9\ in)(96\ in) = 1752\ kip < 1783\ kip$$

Although the strut-to-node interface does not have enough capacity to resist the applied stress according to the calculation above, the percent difference between the applied force and the resistance is less than 2 percent, as computed below:

$$\%\ Difference = \left(\frac{1783\ kip - 1752\ kip}{1783\ kip}\right)(100) = 1.7\%$$

This difference is relatively insignificant, and it is reasonably safe to assume that f'_c will exceed the specified value by 1.7% (only 102 psi). Therefore, the strut-to-node interface is considered to have adequate strength.

Therefore, the strength of each face of Node *E* is sufficient to resist the applied forces.

Node B *(CCT):*

The geometry and forces at Node *B* are shown in Figure 2-15. Its geometry is defined by the effective square bearing area previously calculated in this design example, the location of the tie along the top of the bent cap, and the angle of Strut *BD*. The length of the bearing face of the node is equal to the dimension of the effective square bearing area, or 48.2 inches. As specified in *AASHTO LRFD* Article 5.8.2.5.2, the length of the back face is taken as double the distance from the centroid of the longitudinal reinforcement, or Tie *AB*, to the top face of the bent cap (measured perpendicular to the top face). The length of the strut-to-node interface is computed as follows, where 83.2° is the angle of Strut *BD* relative to the top surface of the cap:

$$w_s = l_b \sin\theta + a\cos\theta = (48.2\ in)\sin 83.2° + (10.3\ in)\cos 83.2° = 47.9\ in + 1.2\ in = 49.1\ in$$

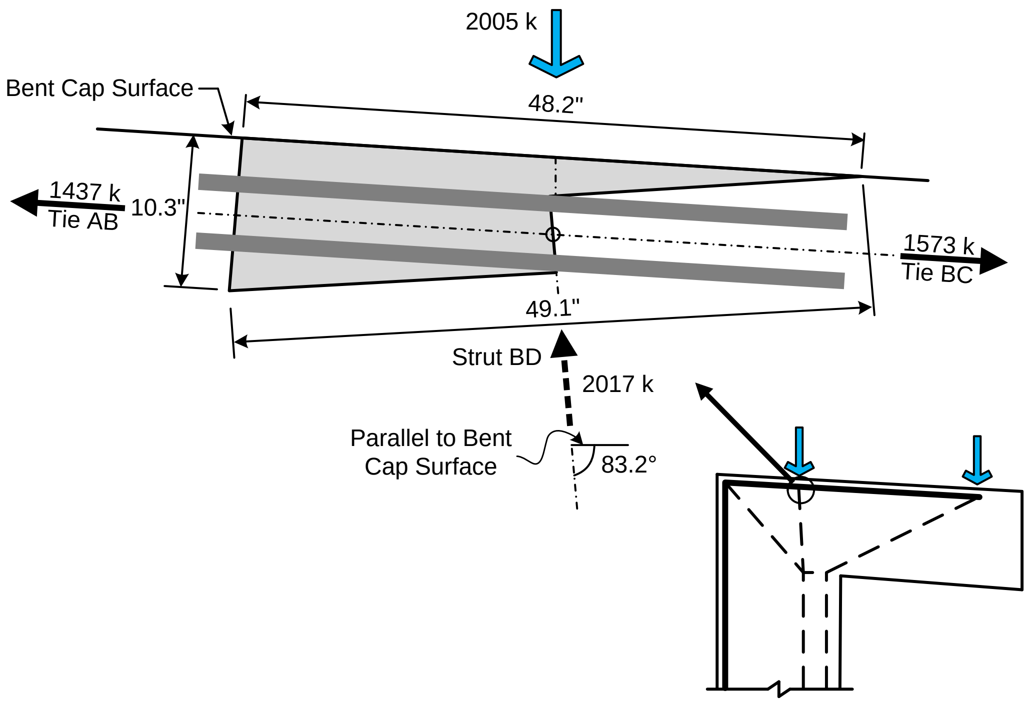


Figure 2-15: Geometry and Forces at Node *B*

The strength of each bearing area at Node *B* (i.e., those supporting Beam 1 and Girder 1) should be checked for adequacy. The size of each bearing pad or plate is summarized in Table 2-1, and the factored load corresponding to each beam or girder is presented in Figure 2-6(a). Since Node *B* is a CCT node (i.e., ties intersect the node in only one direction), a concrete efficiency factor, *v*, of 0.70 is applied to the strengths of the bearings (*AASHTO LRFD* Table 5.8.2.5.3a-1).

In the following computations, the actual bearing areas are used to be conservative. As previously stated, the bearings sit on seats that allow the force to be spread over a larger area. If the following calculations revealed that the node was insufficient to resist the applied forces, the confinement modification factor, *m*, could be applied to the concrete capacity, taking into account the larger spread areas.

Bearing for Beam 1:

Bearing area: $A_{bearing} = 2(16\ in)(9\ in) = 288\ in^2$

Factored load: $P_u = 404\ kip$

Concrete efficiency factor: $\nu = 0.70$

Concrete capacity:

$$f_{cu} = \nu \cdot f'_c = (0.70)(6.0\ ksi) = 4.2\ ksi$$

$$\Phi \cdot P_n = (0.7)(4.2\ ksi)(288\ in^2) = 847\ kip > 404\ kip \quad \boldsymbol{OK}$$

Bearing for Girder 1:

Bearing area: $A_{bearing} = (42\ in)(29.5\ in) = 1239\ in^2$

Factored load: $P_u = 1396\ kip$

Concrete efficiency factor: $\nu = 0.70$

Concrete capacity:

$$f_{cu} = \nu \cdot f'_c = (0.70)(6.0\ ksi) = 4.2\ ksi$$

$$\Phi \cdot P_n = (0.7)(4.2\ ksi)(1239\ in^2) = 3643\ kip > 1396\ kip \quad \boldsymbol{OK}$$

The tie forces at Node *B* result from the anchorage of the reinforcing bars and do not concentrate at the back face. In cases where the back face does not resist a direct force, no back face check is necessary.

The strength of the strut-to-node interface of Node *B* is checked as shown below. As explained with previous computations, the use of the confinement modification factor, *m*, can be used as needed. It is included here for illustrative purposes. A_2 is taken as the width of the cap beam, 96 inches (see Figure 2-1), and A_1 is taken as the dimension of the effective square bearing area, 48.2 inches (see Design Step 3). As specified in *AASHTO LRFD* Article 5.8.2.5.3a, the confinement modification factor, *m*, is computed as follows:

$$m = \sqrt{A_2 / A_1} = \sqrt{(96\ in)^2 / (48.2\ in)^2} = 1.99 < 2$$

$$\therefore \text{Use } m = 1.99$$

Strut-to-node interface:

Factored load: $P_u = 2017\ kip$

Concrete efficiency factor: $\nu = 0.85 - 6.0\ ksi / 20\ ksi = 0.55$

$\therefore$ Use $\nu = 0.55$

Concrete capacity:

$$f_{cu} = m \cdot \nu \cdot f'_c = (1.99)(0.55)(6.0\ ksi) = 6.6\ ksi$$

$$\Phi \cdot P_n = (0.7)(6.6\ ksi)(49.1\ in)(48.2\ in) = 10{,}934\ kip > 2017\ kip \quad \textbf{OK}$$

Therefore, the strength of Node *B* is sufficient to resist the applied forces.

***Node* C (CCT):**

Node *C* is shown in Figure 2-16. The geometry of the node is determined in a manner similar to that of Node *B*. The length of the bearing face of the node, 39.1 inches, was previously calculated in this design example. The following set of checks is similar to that performed for Node *B* (since both nodes are CCT nodes). The length of the strut-to-node interface is computed as follows, where 31.2° is the angle of Strut *CE* relative to the top surface of the cap:

$$w_s = l_b \sin\theta + a\cos\theta = (39.1\ in)\sin 31.2° + (10.3\ in)\cos 31.2° = 20.3\ in + 8.8\ in = 29.1\ in$$

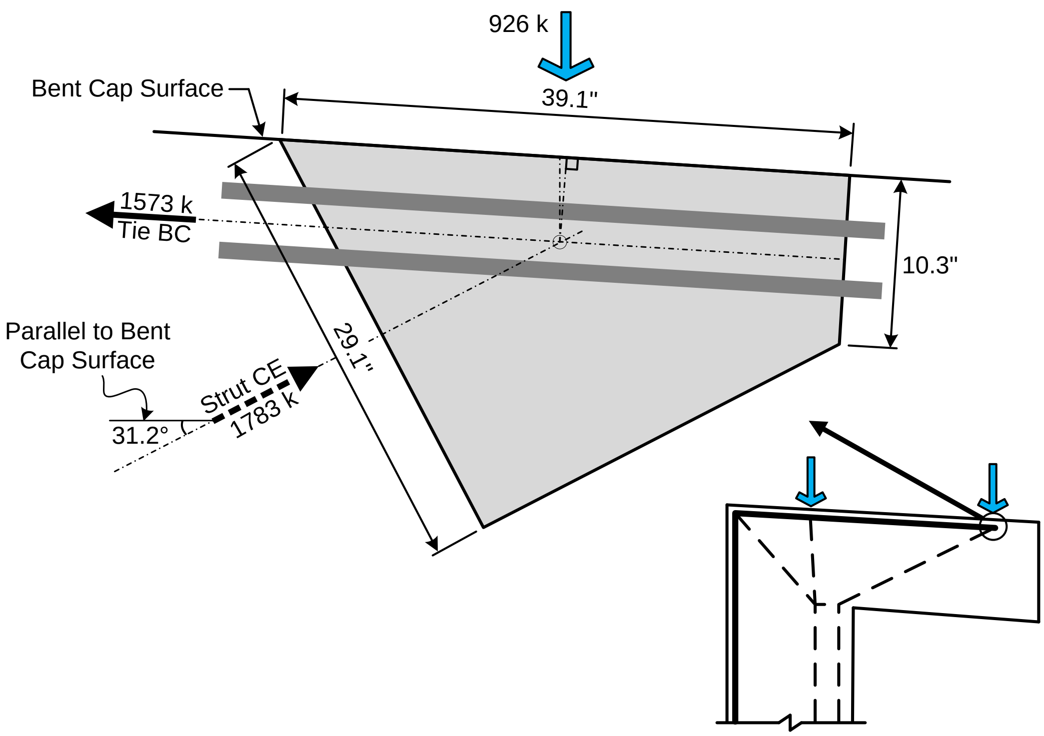


Figure 2-16: Geometry and Forces at Node *C*

Bearing for Beam 2:

Bearing area: $A_{bearing} = 2(16\ in)(9\ in) = 288\ in^2$

Factored load: $P_u = 404\ kip$

The bearing check for Beam 2 is the same as that for Beam 1 and therefore satisfies the requirements.

Bearing for Girder 2:

Bearing area: $A_{bearing} = (24\ in)(24\ in)$
$= 576\ in^2$

Factored load: $P_u = 367\ kip$

Concrete efficiency factor: $\nu = 0.70$

Concrete capacity:

$$f_{cu} = \nu \cdot f'_c = (0.70)(6.0\ ksi) = 4.2\ ksi$$

$$\Phi \cdot P_n = (0.7)(4.2\ ksi)(576\ in^2) = 1693\ kip$$

$$> 367\ kip \quad \boldsymbol{OK}$$

Confinement modification factor (*AASHTO LRFD* Article 5.8.2.5.3a):

$$m = \sqrt{A_2/A_1} = \sqrt{(96\ in)^2/(39.1\ in)^2} = 2.46 > 2$$

$$\therefore \textit{Use } m = 2$$

Strut-to-node interface:

Factored load: $P_u = 1783\ kip$

Concrete efficiency factor: $\nu = 0.85 - 6.0\ ksi/20\ ksi = 0.55$

$\therefore$ Use $\nu = 0.55$

Concrete capacity:

$$f_{cu} = m \cdot \nu \cdot f'_c = (2)(0.55)(6.0\ ksi) = 6.6\ ksi$$

$$\Phi \cdot P_n = (0.7)(6.6\ ksi)(29.1\ in)(39.1\ in) = 5257\ kip$$

$$> 1783\ kip \quad \boldsymbol{OK}$$

Therefore, the strength of Node *C* is sufficient to resist the applied forces.

Node* A *(CTT – Curved-Bar Node):

Node *A* is a curved-bar node, located in the top left corner of the cantilever bent cap. As specified in *AASHTO LRFD* Article C5.8.2.2, curved-bar node checks are not required. Therefore, a check of Node *A* is not presented in this design example.

Node* D *(CCC):

Node *D* is an interior node with no bearing plate or geometrical boundaries to clearly define its geometry. It is therefore a smeared node. As specified in *AASHTO LRFD* Article C5.8.2.2, a check of concrete stresses in smeared nodes is unnecessary. Therefore, a check of Node *D* is not presented in this design example.

Design Step 9 - Proportion Crack Control Reinforcement

Crack control reinforcement design provisions are presented in *AASHTO LRFD* Article 5.8.2.6. Since crack control reinforcement was compared with proportioning of the ties, it was computed as part of Design Step 7. Based on those computations, the following reinforcement will be used in the bent cap:

- Use 4 legs of #6 stirrups with spacing less than 6.1 inches within the cantilevered portion of the bent cap
- Use 2 legs of #8 stirrups with spacing less than 5.5 inches above the column
- Use #8 bars with spacing less than 5.5 inches as horizontal crack control reinforcement

Final reinforcement details are provided in Figures 2-18, 2-19, and 2-20.

Design Step 10 - Provide Necessary Anchorage for Ties

Anchorage at Node C*:*

As specified in *AASHTO LRFD* Articles 5.8.2.4.2 and C5.8.2.4.2, the primary longitudinal reinforcement of the cantilever must be properly developed at Node *C*. The available length for the development of the tie bars is measured from the point where the centroid of the reinforcement enters the extended nodal zone (assuming the diagonal strut is prismatic) to the tip of the cantilever, leaving the required clear cover, as illustrated in Figure 2-17.

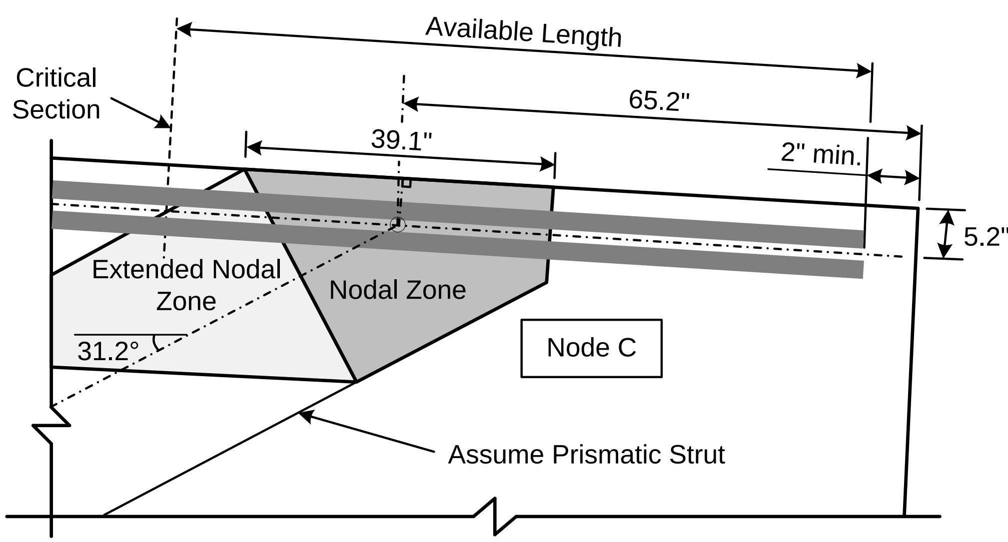


Figure 2-17: Anchorage of Longitudinal Bars at Node *C*

Providing 2 inches of clear cover, the available length for the primary longitudinal reinforcement of the cantilever (measured at the centroid of the bars) is computed as follows:

$$\text{Available length} = 65.2\ in + {}^{39.1\ in}/_{2} + {}^{5.2\ in}/_{tan31.2°} - 2\ in = 91.3\ in$$

Each of the dimensional values in the above calculation is shown in Figure 2-17. As specified in *AASHTO LRFD* Article 5.10.8.2.1a, the straight development length is then computed as follows:

$$l_d = \frac{2.4 d_b f_y}{\sqrt{f'_c}} \cdot 1.3 = \frac{2.4(1.41\ in)(60\ ksi)}{\sqrt{6.0\ ksi}} \cdot 1.3 = 108\ in > 91.3\ in$$

The 1.3 factor in the above equation is a modification factor to account for more than 12 inches of fresh concrete cast below the reinforcement (*AASHTO LRFD* Article 5.10.8.2.1b). In addition, *AASHTO LRFD* Article 5.10.8.2.1c presents modification factors that decrease the development length.

Based on these computations, sufficient straight-bar anchorage is not provided for the tie at Node *C*, and hooks are used to provide sufficient anchorage. Design requirements for standard hooks in tension are provided in *AASHTO LRFD* Article 5.10.8.2.4 and are not presented here.

Splice between Cantilever and Column Reinforcement:

In addition to ensuring adequate anchorage of the tie bars, a splice is designed between the primary longitudinal reinforcement of the cantilever and the main column tension reinforcement. All 20 longitudinal reinforcing bars will be spliced, and the ratio of the area of the steel provided to the area required is less than 2. The splice is therefore a Class B splice with a required length of $1.3l_d$ (*AASHTO LRFD* Article 5.10.8.4.3a), calculated as follows:

$$1.3 l_d = 1.3 \cdot \frac{2.4 d_b f_y}{\sqrt{f'_c}} \cdot 1.3 = 1.3 \cdot \frac{2.4(1.41\ in)(60\ ksi)}{\sqrt{6.0\ ksi}} \cdot 1.3 = 140\ in$$

The required splice length must be provided within the depth of the cap and the top portion of the column.

Design Step 11 - Draw Reinforcement Layout

The reinforcement details for the load case considered in this design example are presented in Figures 2-18, 2-19, and 2-20. Any reinforcement details shown in these figures that were not previously described within this design example can be adjusted based on the specific state or agency policies and practices.

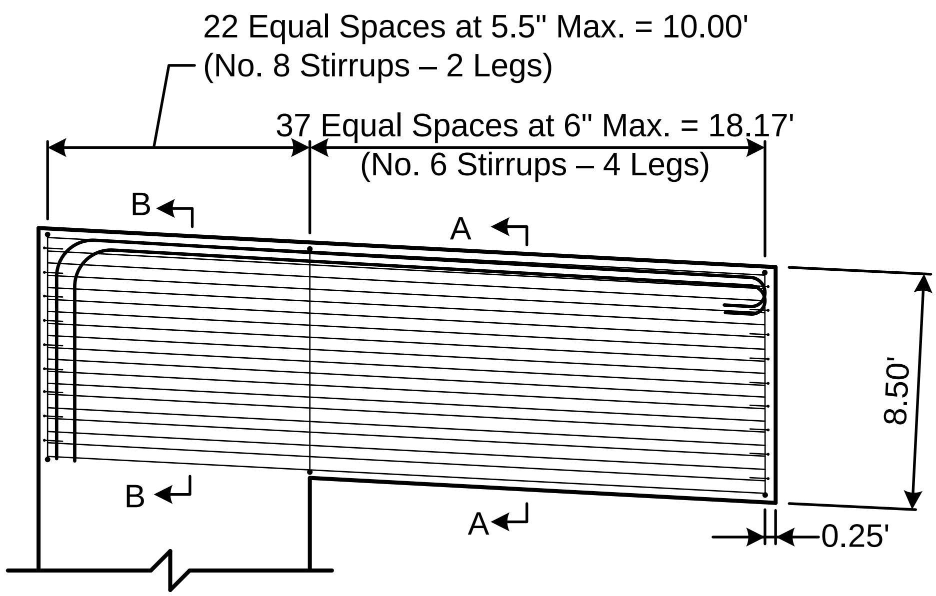


Figure 2-18: Elevation View of Reinforcement Details (Based on STM Specifications)

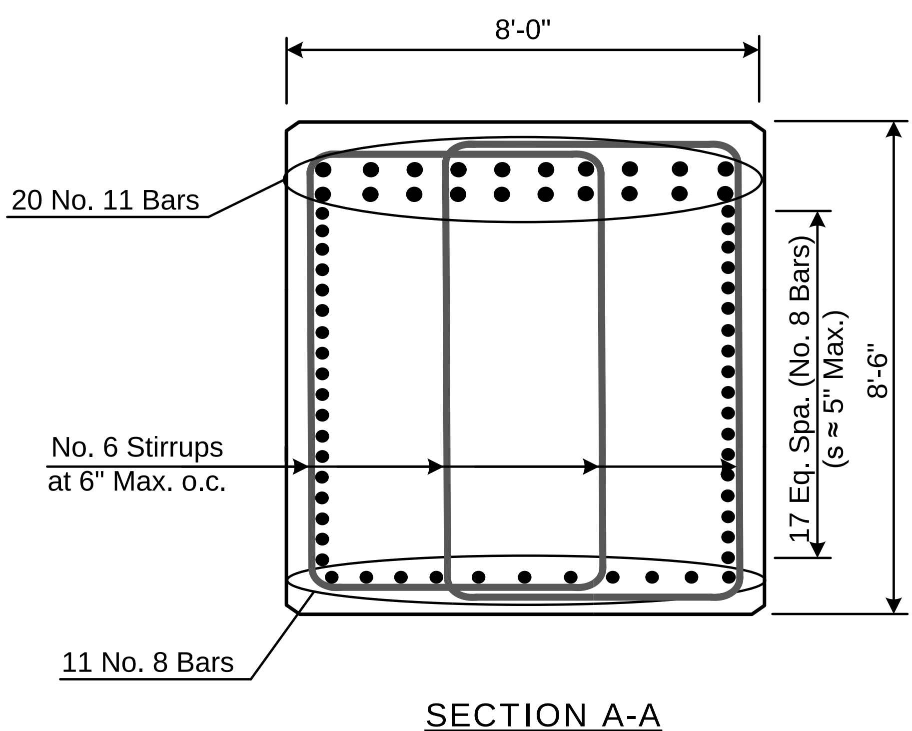


Figure 2-19: Section A-A Showing Reinforcement Details (Based on STM Specifications)

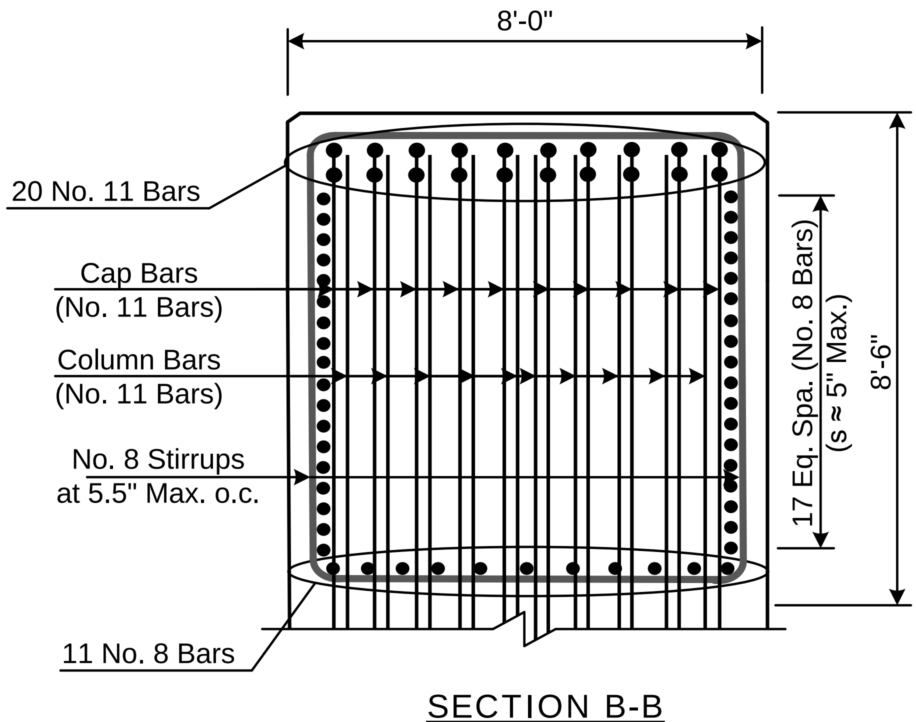


Figure 2-20: Section B-B Showing Reinforcement Details (Based on STM Specifications)

References:

1. Birrcher, D.B., Tuchscherer, R.G., Huizinga, M.R., Bayrak, O., Wood, S.L., and Jirsa, J.O., *Strength and Serviceability Design of Reinforced Concrete Deep Beams*, Technical Report 0-5253-1, Center for Transportation Research, Bureau of Engineering Research, University of Texas at Austin, April 2009, 376 pp.

2. Williams, C.S., Deschenes, D.J., and Bayrak, O., *Strut-and-Tie Model Design Examples for Bridges*, Implementation Report 5-5253-01, Center for Transportation Research, Bureau of Engineering Research, University of Texas at Austin, October 2011, 272 pp.

Design Example #3 – Inverted-Tee Moment Frame Straddle Bent Cap

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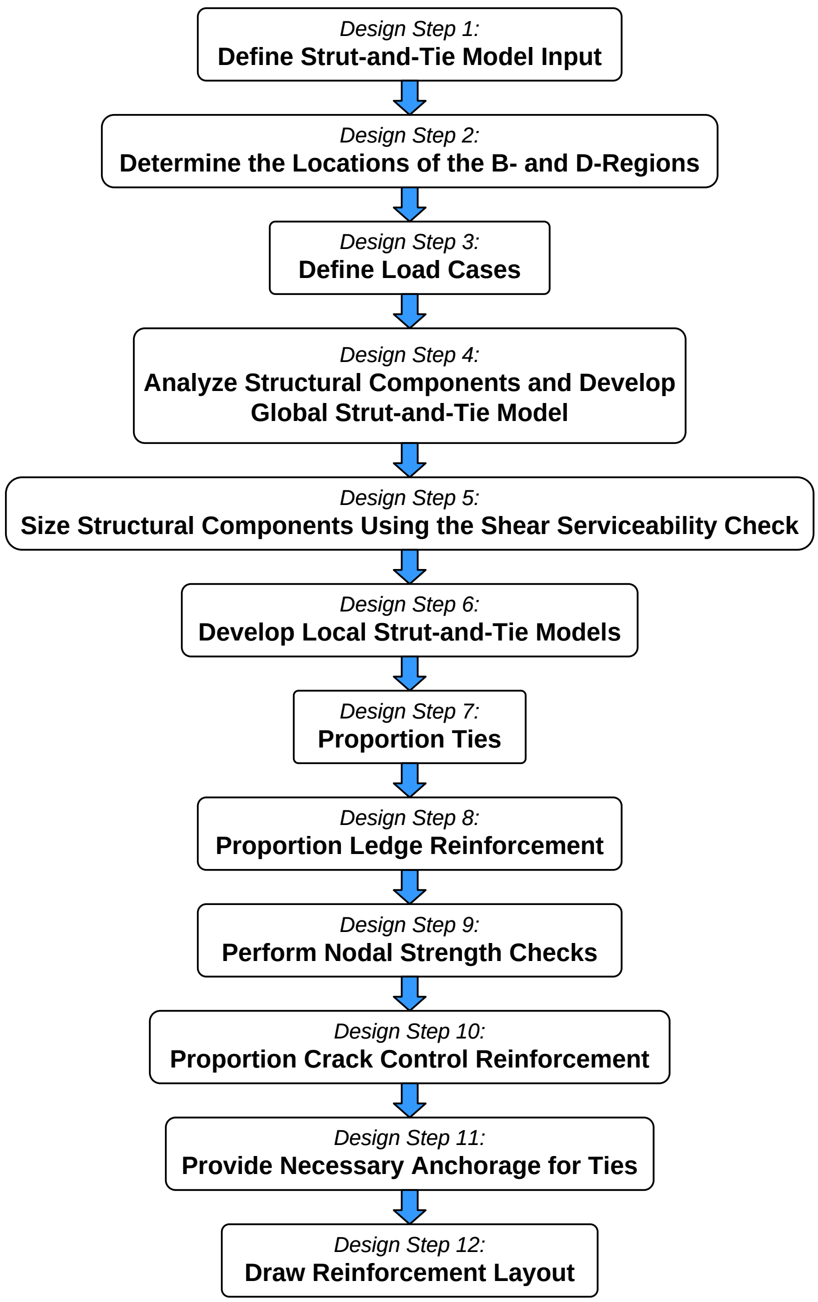
Design Example #3 presents the application of the strut-and-tie method (STM) to the analysis and design of an inverted-tee bent cap beam. A complete step-by-step design is presented for one of multiple load cases that must be considered in the design of a structure of this type. The inverted-tee bent cap beam is part of a moment frame straddle bent, which will carry a flyover ramp over a highway below. This design example requires the use of global and local strut-and-tie models to fully model the flow of forces within the cap beam. The example features the elements of strut-and-tie design of concrete members listed below:



Please note that this example is based on an example problem developed in an implementation project sponsored by TxDOT (Report No. 5-5253-01, Williams et al., 2011). Figures included in this design example are adapted from this report. The example has been revised herein to provide additional explanations and to provide compliance with the STM provisions of the 8th edition of *AASHTO LRFD*, as appropriate.

Design Step 1 - Define Strut-and-Tie Model Input

Elevation and plan views of the moment frame straddle bent are shown in Figure 3-2. The straddle bent supports three trapezoidal box beams of a flyover ramp using neoprene bearing pads, which rest on the ledges of the inverted-tee straddle bent cap beam. The bent cap is 47.50 ft long. A depth of 6.00 ft is used for this design example. The stem of the cap beam is 3.34 ft wide with 1.33 ft wide ledges projecting from each side, resulting in a total beam width of 6.00 ft at the ledges. The cap beam is supported by two 5.00 ft by 3.00 ft rectangular columns.

The cap beam has a cross-slope to accommodate the superelevation of the curved flyover ramp. In this example, the cross-slope is deemed insignificant to the design of the beam; therefore, a simplified orthogonal model will be used for design. Designing the cap beam as sloped or orthogonal may be valid (dependent on the cross-slope of the beam); therefore, the designer should use his/her discretion to decide which approach is most appropriate.

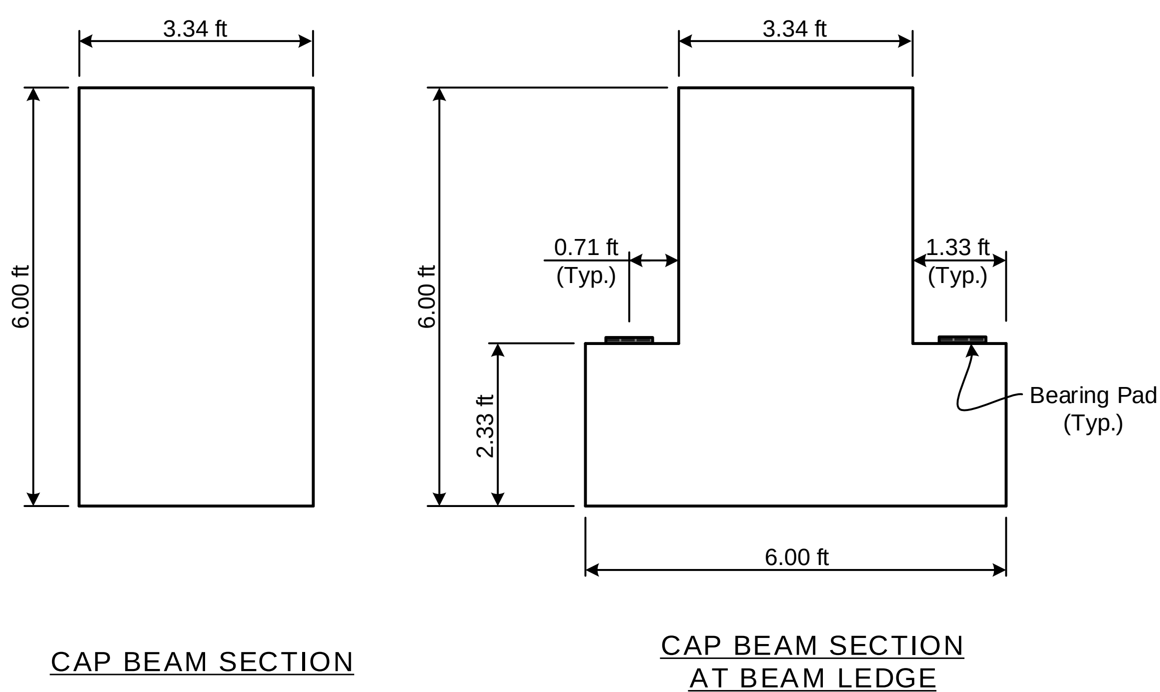


Figure 3-1: Typical Sections of Straddle Bent Cap Beam

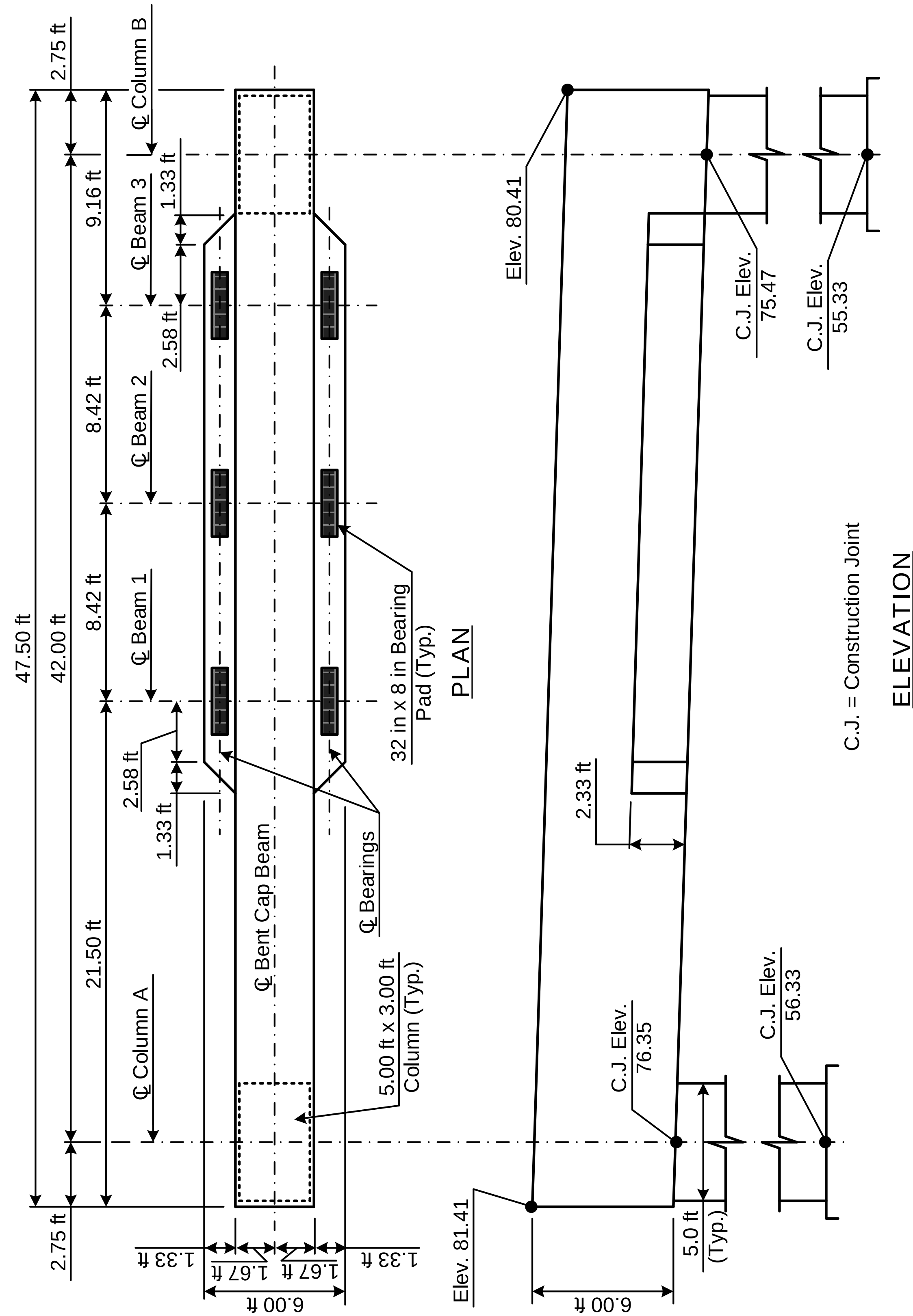


Figure 3-2: Inverted-Tee Beam Straddle Bent for Design Example 3

The design compressive strength of the concrete, f'_c, is taken as 6.0 ksi, and the yield strength of the steel reinforcing, f_y, is taken as 60.0 ksi. Six trapezoidal box beams each rest on a rectangular 34.0 in by 8.0 in neoprene bearing pad on the cap beam ledges. The bearing stresses from the bearing pads may be assumed to spread laterally through the bearing seats; however, for simplicity this effect will be ignored in this design example.

Design Step 2 - Determine the Locations of the B- and D-Regions

The subject straddle bent cap beam in this design example contains numerous disturbances, including the superstructure loads, the beam ledges, and the corners of the moment frame itself. Referring to Figure 3-3, although a small region of the cap beam may be considered a B-Region, the entire cap beam is more appropriately designed using the STM procedures.

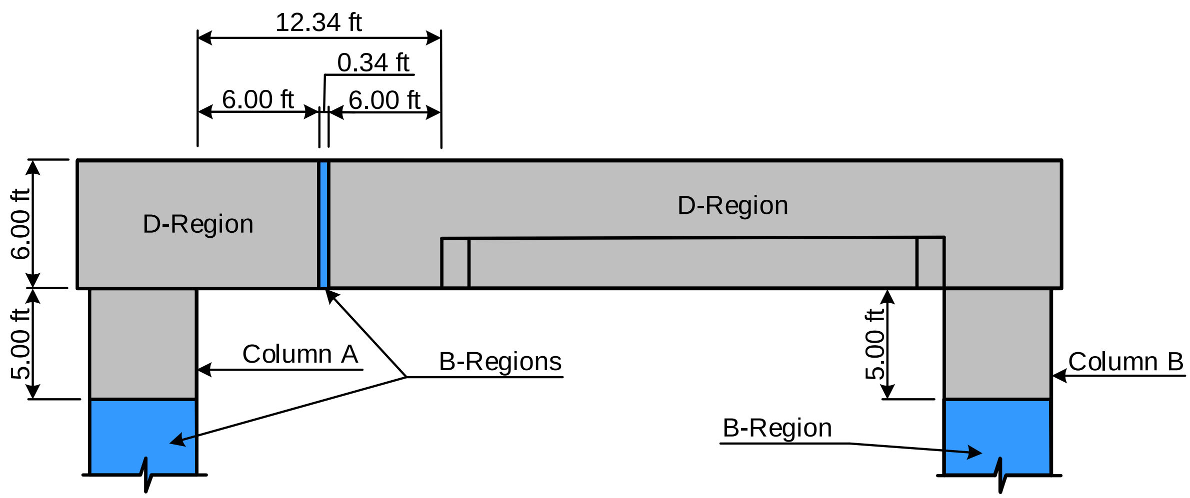


Figure 3-3: Bent Divided into B- and D-Regions

Design Step 3 - Define Load Cases

Factored beam loads applied to the cap beam are shown in Figure 3-4. The loads are applied symmetrically about the centerline of the cap beam. This load case is only one load case that must be considered during final design. Additional load cases should be evaluated to determine if other load cases govern the design of certain strut-and-tie members.

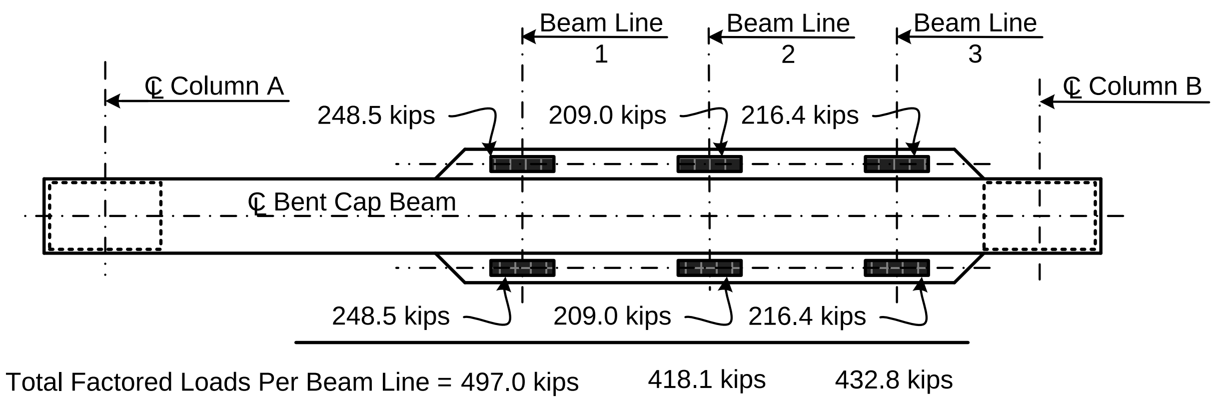


Figure 3-4: Factored Beam Loads per Beam Line

In addition to the factored beam loads, the factored self-weight load of the beam cap must be determined. The self-weight loads are determined using a unit weight of concrete of 150 lb/ft^3. A load factor of 1.25 is applied to the self-weight in accordance with the *AASHTO LRFD* Strength I load combination.

Since the self-weight of the cap beam must eventually be distributed to each of the nodes in the STM truss model, the magnitude of each of the self-weight nodal loads is dependent on the actual geometry of the STM. A diagram of the self-weight loads acting on the bent cap beam is given in Figure 3-5 on the following page. The uniform dead loads of the basic (rectangular) beam and the beam ledge will be resolved into point loads for application to the strut-and-tie truss.

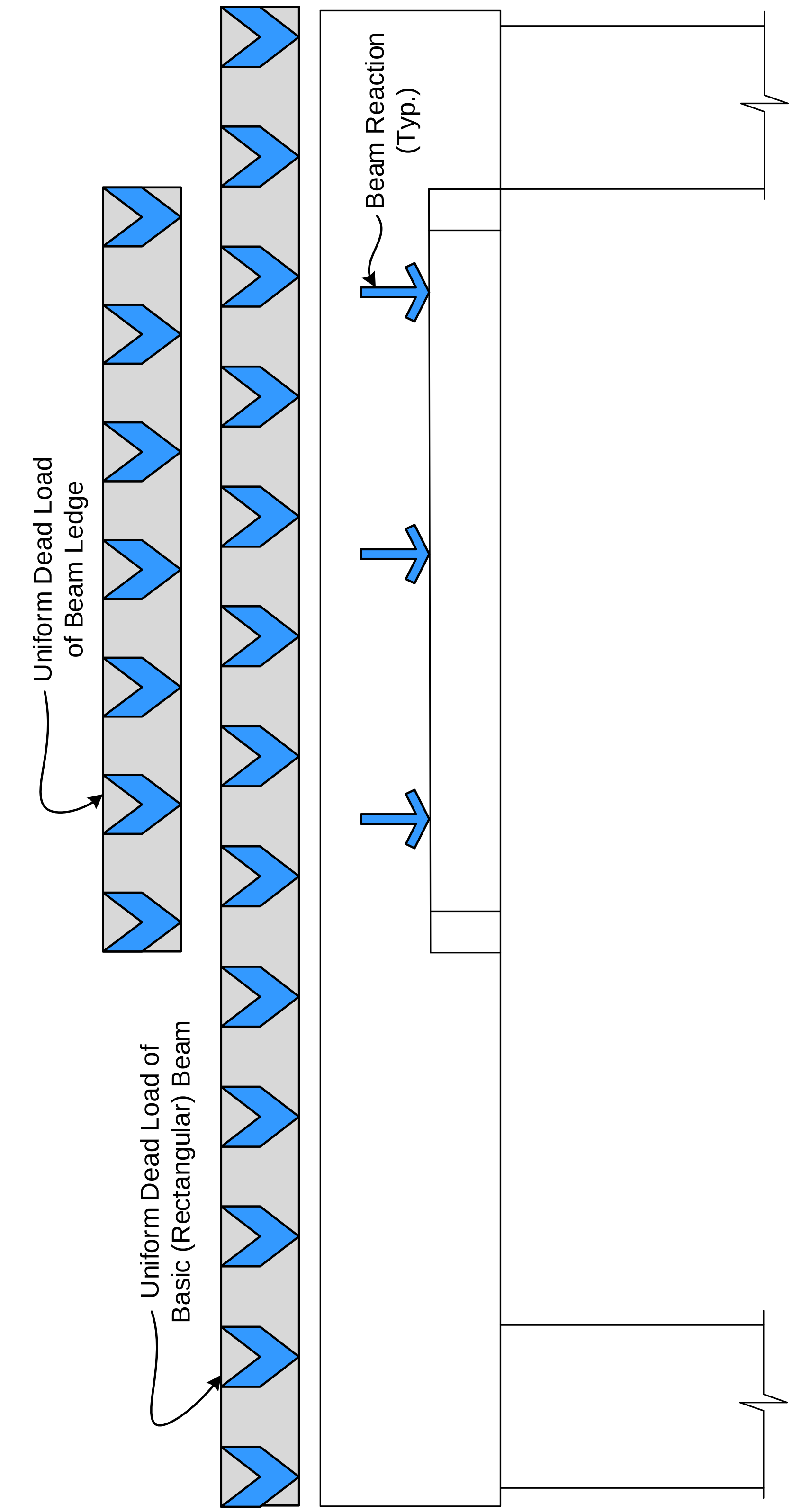


Figure 3-5: Loads Acting on the Global Strut-and-Tie Model

Design Step 4 - Analyze Structural Components and Develop Global Strut-and-Tie Model

Design of the straddle bent cap beam requires the use of global and local strut-and-tie models to effectively model the flow of forces within the cap beam. The global model is used to model the flow of forces from the beam bearing pads to the supporting columns. The local strut-and-tie models illustrate the flow of forces around the cross section of the cap beam, which are used to design the beam ledges. The global and local models together form a three-dimensional strut-and-tie model of the cap beam.

Before determining the geometry of the global strut-and-tie model, an analysis of the moment frame bent itself must be performed to determine the structure reactions to the externally-applied loads. Each frame member in this analysis is located at the center of gravity of its respective cross-section. A constant flexural stiffness is assumed for the cap beam, and both columns are modeled as 5 ft by 3 ft rectangular sections.

Modeling the Cap Beam:

With the widespread availability of finite element analysis software today, it is relatively easy to model the cap beam using the actual sections of the cap beam. Correctly modeling the cap beam stiffness will improve the strut-and-tie model and is recommended. The simplified model of the cap beam used in this example allows the example moment frame analysis to be performed by hand as a check.

The internal forces in the moment frame determined in Figure 3-6 are used to estimate the locations of the struts and ties in the bent columns. The uniform cap beam load and uniform beam ledge load are resolved into point loads and applied to the moment frame. The loads are applied where the estimated STM truss nodes will be located.

Single Analysis vs. Multiple Analyses:

The approach shown in this design example uses a single analysis of the moment frame; assuming the locations and magnitudes of the cap beam self-weight loads and analysis of the moment frame are performed in a single step. This approach gives reasonable estimates of the cap beam forces.

The moment frame analysis may be improved by performing the analysis in multiple steps. First, only the external beam loads are applied to the moment frame, and the frame reactions and internal forces are determined. The locations of the vertical struts and ties in the frame columns are then located based on the results of this analysis. The global strut-and-tie model of the straddle bent cap beam is then defined. The factored beam self-weights are then calculated by tributary volumes and distributed to each of the nodes in the STM truss. The moment frame analysis is then performed again to eliminate discrepancies between the moment frame internal forces and the member forces in the STM truss. The strut and tie locations may then be adjusted and the truss re-analyzed. Iterating in this manner will improve the results of the analysis.

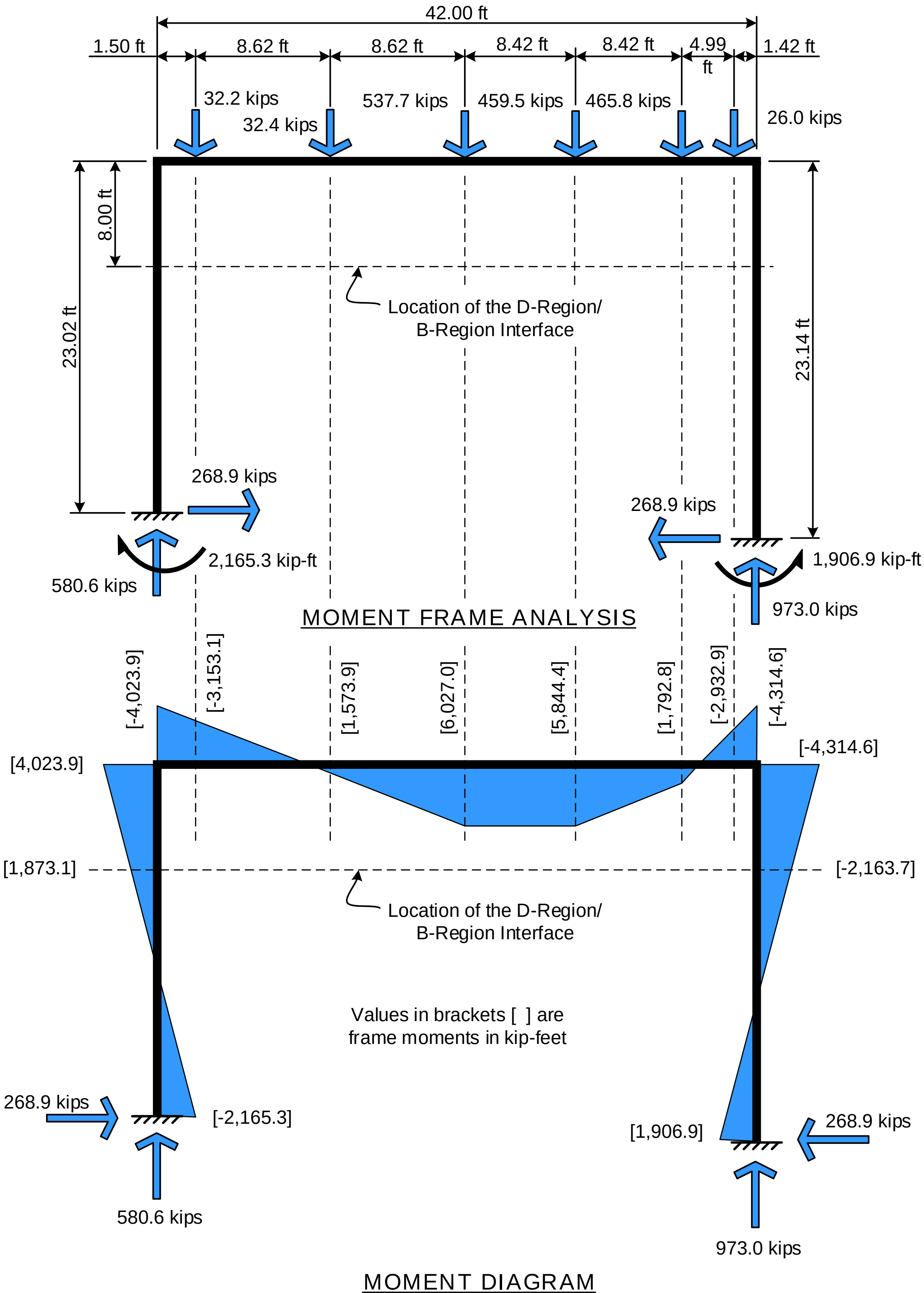


Figure 3-6: Moment Frame Analysis and Resulting Moment Diagram

The locations of the column struts are based on the results of the moment frame analysis in Figure 3-6. The distribution of stress within the column is assumed to be linear at a point equal to one member depth below the bottom of the cap beam (i.e. at the B-Region/D-Region interface). The internal forces at the interface are shown below in Figure 3-7.

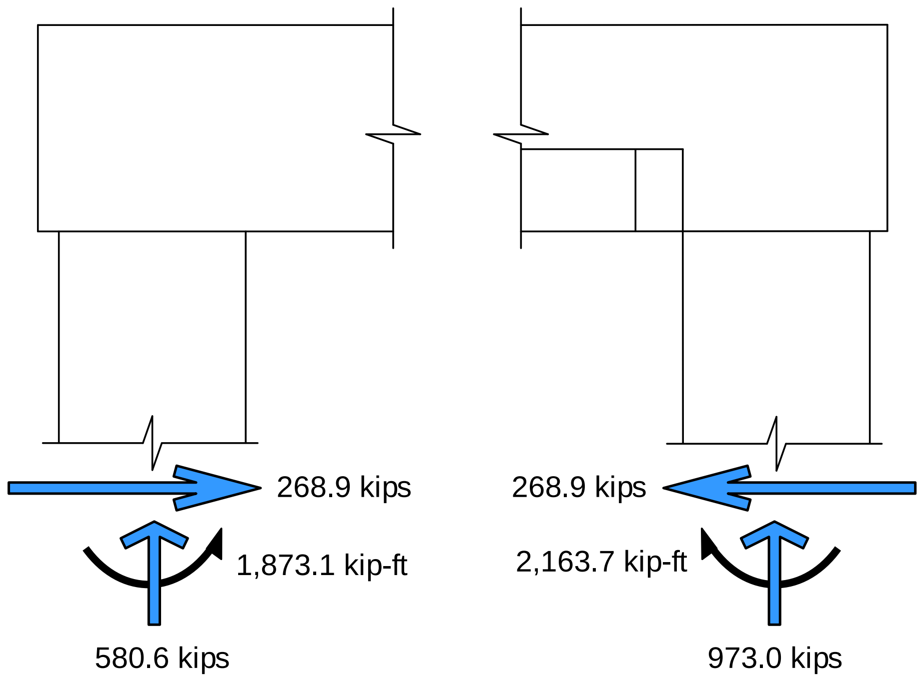


Figure 3-7: Moment Frame Forces at B-Region/D-Region Interface

Next, the stress distributions at the interface must be determined in order to locate the column struts and ties. These stress distributions will be assumed to be linear (note that in reality, the stress will probably not be linear because of cracking of the concrete). The area and moment of inertia of the columns are first calculated:

$$A_{column} = 3.00\ ft\ \times 5.00\ ft = 15.00\ ft^2$$

$$I_{column} = \frac{bh^3}{12} = \frac{3.00\ ft \times (5.00\ ft)^3}{12} = 31.25\ ft^4$$

Next, determine the stresses at each face of each column. Compressive stress will be taken as negative. Beginning with the left column (Column A):

$$f_{Column\ A} = \frac{P}{A} \pm \frac{My}{I} = \frac{-580.6\ kips}{15.00\ ft^2} \pm \frac{1{,}873.1\ kip-ft \times 2.50\ ft}{31.25\ ft^4}$$

$$f_{Column\ A} = \begin{cases} -38.7 + 149.8 = 111.1\ \frac{kip}{ft^2} = 0.77\ \frac{kip}{in^2}\ (Tension) \\ -38.7 - 149.8 = -188.5\ \frac{kip}{ft^2} = -1.31\ \frac{kip}{in^2}\ (Compression) \end{cases}$$

The location of the compression force resultant may now be obtained by using similar triangles:

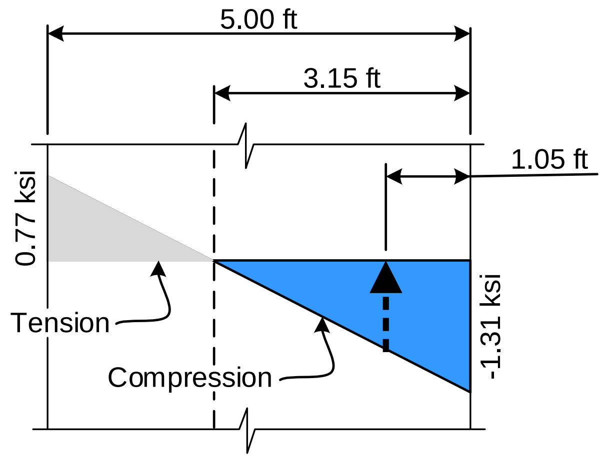


Figure 3-8: Stress Distribution in Column A

By similar triangles, the neutral axis is located at 3.15 ft from the right face of Column A. The compression strut in the column will then be located at the centroid of the triangular compression area. Therefore, the compression strut will be placed at 1.05 ft from the right face of Column A.

Similarly, determine the stresses in the right column (Column B):

$$f_{Column\ B} = \frac{P}{A} \pm \frac{My}{I} = \frac{-973.0\ kips}{15.00\ ft^2} \pm \frac{2{,}163.7\ kip-ft \times 2.50\ ft}{31.25\ ft^4}$$

$$f_{Column\ B} = \begin{cases} -64.9 + 173.1 = 108.2\ \frac{kip}{ft^2} = 0.75\ \frac{kip}{in^2}\ (Tension) \\ -64.9 - 173.1 = -238.0\ \frac{kip}{ft^2} = -1.65\ \frac{kip}{in^2}\ (Compression) \end{cases}$$

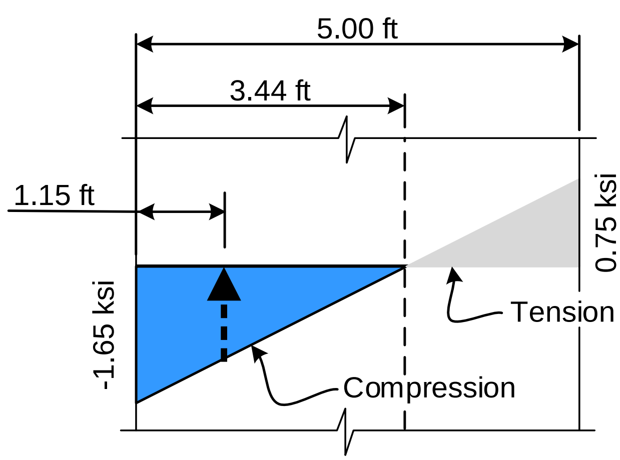


Figure 3-9: Stress Distribution in Column B

By similar triangles, the neutral axis is located at 3.44 ft from the left face of Column B. The compression strut will be located at 1.15 ft from the left face of Column B (approximately one-third of 3.44 ft).

The following discussion details the development of the global STM of the moment frame. For the arrangement of the STM truss, refer to Figure 3-10. Figure 3-10 also includes the individual truss member forces calculated by a truss analysis.

Now that the locations of the column struts have been determined, the ties may be located. The ties will be located at the center of gravity of the longitudinal column reinforcement. This location is assumed to be 3.8 in (0.32 ft) from the outer face of the columns, which allows space for the column stirrups/ties and clear cover.

Next, the locations of the chords of the global truss model must be located. Positive and negative moment regions will exist within the straddle bent beam, requiring ties in both the top and bottom chords. Therefore, the truss chords will be located at the centers of gravity of the longitudinal reinforcement in the straddle bent cap beam. In Figure 3-10, the top chord is located at 4.6 in (0.38 ft) from the top of the cap beam and the bottom chord is located 6.0 in (0.50 ft) from the bottom of the cap beam. Note that this is a departure from the previous design examples, where the depth of the compression block, *a*, of an analogous Bernoulli beam was used to locate the struts. The resulting truss depth (in the cap beam) is 5.12 ft.

Next, vertical Ties *CI*, *DJ*, and *EK* are placed at the positions of the applied superstructure loads. These ties represent the reinforcement required to “hang” the loads applied to the ledges of the inverted-tee beam and transfer the stress from the beam ledges to the components of the global strut-and-tie model truss.

Recall that the angle between a tie and an adjacent strut should not be less than 25 degrees, as stipulated in *AASHTO LRFD* Article 5.8.2.2. In order to meet this requirement, Tie *BH* is placed halfway between Nodes *G* and *I*. Note that all of the struts are oriented such that they will be in compression. This is also the location of the assumed cap beam self-weight applied in the moment frame analysis.

Continuing, the total factored loads from each beam line are applied to bottom chord Nodes *I*, *J*, and *K*. The factored self-weight loads from the moment frame analysis are distributed to each of the nodes in the STM truss, except Nodes *A* and *F*. Because of their location at the upper corners of the truss, applying any self-weight at these nodes would be unreasonable.

To account for the shears in the frame at the D-Region/B-Region interfaces, Ties *A’G’* and *L’F’* are added at the base of the STM truss to accept the horizontal shear loads and Struts *A’G* and *F’L* are added to anchor Ties *A’G’* and *L’F’*.

Subdividing Nodes and Struts:

Keep in mind that nodes with multiple struts entering the nodal zone (i.e. Nodes *C, G,* and *L*) must be subdivided per *AASHTO LRFD* Commentary C5.8.2.2-4. Subdivision of the nodes will addressed during the node design checks.

Additionally, recall that in Design Example 2 (Cantilever Bent Cap) two vertical struts were used to carry the compressive force in the bent column. In this design example, the column struts will similarly be subdivided. Using only one strut in each column simplifies the truss analysis. However, an STM truss with 2 struts in each column may be solved reasonably quickly using a structural analysis software package.

The column reactions that are applied to the strut-and-tie model at the B-Region/D-Region interface are found using the results of the moment frame analysis in Figure 3-7. This is done such that the forces in Ties *AA'* and *FF'* and Struts *GG'* and *LL'* are in equilibrium with the internal forces in each column. The bending moments at the D-Region/B-Region interface are found to be 1,873.1 kip-ft and 2,163.7 kip-ft in Columns A and B, respectively. The axial compressions in Columns A and B are found to be 580.6 kips and 973.0 kips, respectively. In order to determine the strut and tie forces in each column, two systems of two simultaneous equations are solved for the strut and tie forces. The first equation in each system ensures static equilibrium with respect to the axial force in each column. The second equation in each system equates the moments about the centerline of each column to the column bending moment from the moment frame analysis.

Taking forces that are vertical "up" as positive, the system of simultaneous equations for Column A (the left column) is:

$$\begin{cases} -F_{Tie} & +F_{Strut} & = 580.6\ kips \\ 2.18\ ft \times F_{Tie} & +1.45\ ft \times F_{Strut} & = 1{,}873.1\ kip-ft \end{cases}$$

Note that the distances 2.18 ft and 1.45 ft are the distances from the tie and strut to the centerline of the column, respectively.

Solving:

$$\begin{cases} F_{Tie} = 284.1\ kips \\ F_{Strut} = 864.7\ kips \end{cases}$$

Similarly, the system of simultaneous equations for Column B (the right column) is:

$$\begin{cases} -F_{Tie} & +F_{Strut} & = 973.0\ kips \\ 2.18\ ft \times F_{Tie} & +1.35\ ft \times F_{Strut} & = 2{,}163.7\ kip-ft \end{cases}$$

Solving:

$$\begin{cases} F_{Tie} = 240.8\ kips \\ F_{Strut} = 1{,}213.8\ kips \end{cases}$$

Now that the forces in each of the column struts and ties are known, the remaining forces in the strut-and-tie model may be found using simple statics or by using a structural analysis software package.

Solving the STM Truss:

The strut-and-tie model truss may be solved manually by applying the equations of statics, using either the method of joints or the method of sections. If the truss will be solved using a structural analysis software, the forces in the column struts and ties should be imposed on those members for the analysis. This forces equilibrium between the moment frame analysis (beam theory) results and the assumed truss shape.

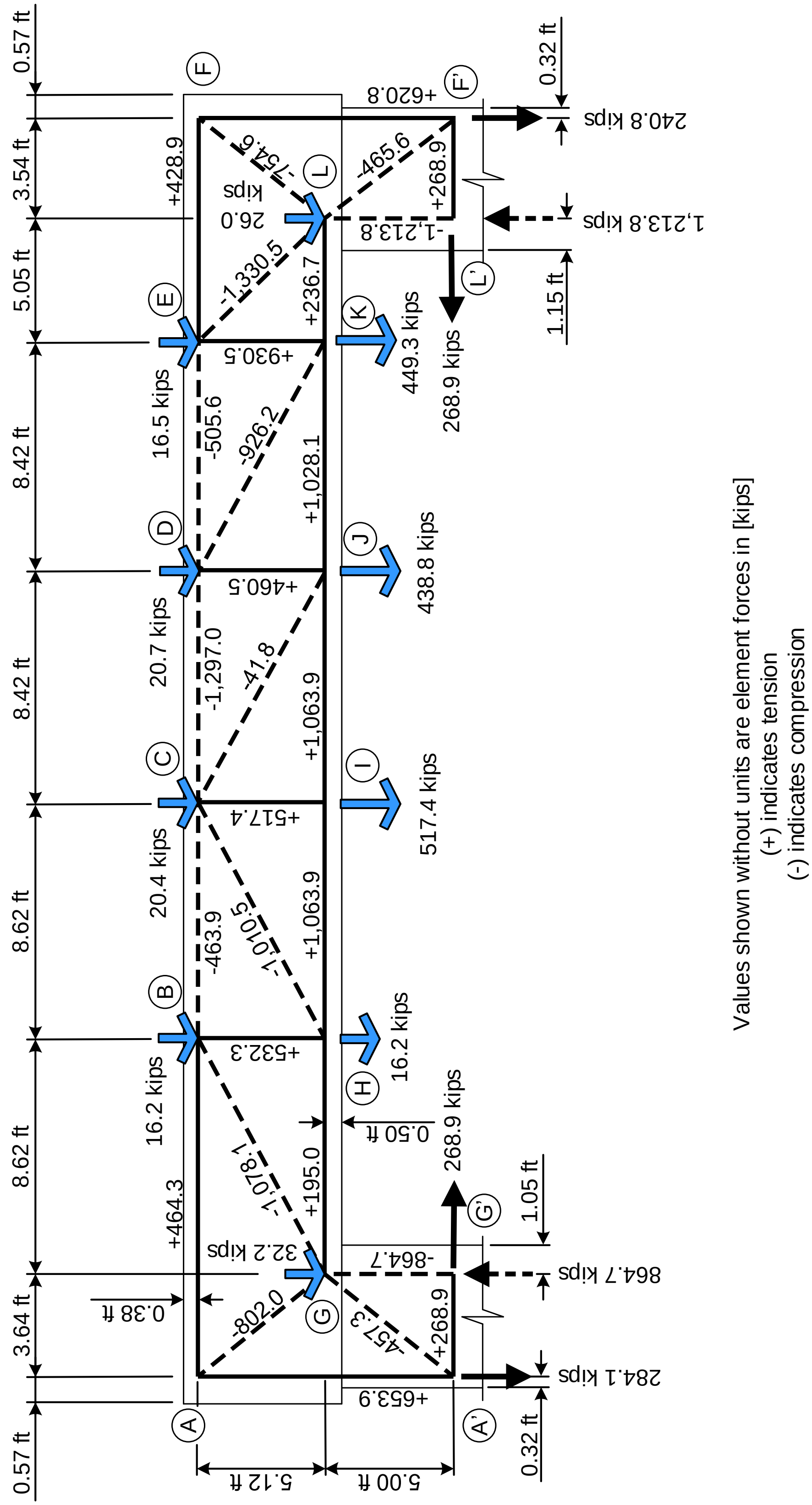


Figure 3-10: Global Strut-and-Tie Model for the Inverted-Tee Cap Beam

Design Step 5 - Size Structural Components Using the Shear Serviceability Check

In Design Step 1, the depth of the cap beam was chosen as 6.00 ft. It was chosen to apply the strut-and-tie design method to the entire cap beam, even though there is a small region within the beam that may be considered a B-Region. The beam stem width must now be verified.

AASHTO LRFD Equation C5.8.2.2-1 limits the applied shear to the following value, V_{cr}, with corresponding minimum and maximum values:

$$V_{cr} = \left[0.2 - 0.1\left(\frac{a}{d}\right)\right]\sqrt{f_c'}\,b_w d$$

limited as follows:

$$0.0632\sqrt{f_c'}\,b_w d \leq V_{cr} \leq 0.158\sqrt{f_c'}\,b_w d$$

As in previous examples, this equation is used to estimate the shear at which diagonal cracks form in D-Regions. Where the applied service load shears are less than V_{cr}, reasonable assurance is provided that diagonal shear cracks will not form. Recall that this check is performed at the *AASHTO LRFD* Service I load combination.

After performing an elastic analysis, the maximum shear force is found near the right end of the bent cap beam. The Service I shear at Column B (the right column) is found to be:

$$V_{Service} = 675.9\ kips$$

Therefore, the shear serviceability check will determine the risk of crack formation in the shear span between Beam Line 3 and the centerline of Strut *LL'*. The shear span, *a*, is taken as the horizontal distance between Nodes *K* and *L*, which is 60.6 in, or 5.05 ft.

Because the moment in this area of the cap beam is negative, the distance to the tension reinforcement, *d*, is calculated from the bottom face of the cap beam to the centroid of the top reinforcing steel:

$$d = 72.0\ in - 4.6\ in = 67.4\ in$$

Now that all quantities are known, V_{cr} may now be calculated:

$$V_{cr} = \left[0.2 - 0.1\left(\frac{a}{d}\right)\right]\sqrt{f_c'}\,b_w d$$

First, check the limits on the term: $\left[0.2 - 0.1\left(\frac{a}{d}\right)\right]$

$$\left[0.2 - 0.1\left(\frac{a}{d}\right)\right] = \left[0.2 - 0.1\left(\frac{60.6\ in}{67.4\ in}\right)\right] = 0.110$$

This value is less than the upper limit of 0.158 and greater than the lower limit of 0.0632, therefore use the value of 0.110:

$$V_{cr} = 0.110 \times \sqrt{6.0\ ksi} \times 40.0\ in \times 67.4\ in = 726.4\ kips$$

$$726.4\ kips > 675.9\ kips \quad \boldsymbol{OK}$$

In addition, it is wise to check the left end of the cap beam because of the longer shear span. The shear span in this region is:

$$a = 8.62\ ft \times \frac{12\ in}{ft} = 103.4\ in$$

The maximum service load shear in this shear span is found to be:

$$V_{Service} = 388.6\ kips$$

First, check the limits on the term: $\left[0.2 - 0.1\left(\frac{a}{d}\right)\right]$

$$\left[0.2 - 0.1\left(\frac{a}{d}\right)\right] = \left[0.2 - 0.1\left(\frac{103.4\ in}{67.4\ in}\right)\right] = 0.047$$

Since 0.047 < 0.0632, use the lower bound value of 0.0632:

$$V_{cr} = 0.0632 \times \sqrt{6.0\ ksi} \times 40.0\ in \times 67.4\ in = 417.4\ kips$$

$$417.4\ kips > 388.6\ kips \quad \boldsymbol{OK}$$

Therefore, both checks are satisfactory and the designer should not expect diagonal cracking at service loads. The values of V_{cr} should also be checked using other load cases and if the value of *d* varies significantly from the assumed value of 67.4 in.

Design Step 6 - Develop Local Strut-and-Tie Models

Since the flow of forces in the inverted-tee bent cap beam is very complex, separate strut-and-tie models should be developed at each location where a beam load is supported by the bent cap beam ledge. The strut-and-tie model for a section cut at Beam Line 1 is shown in Figure 3-11 on the next page. Ties A_sG_s and B_sH_s are placed to coincide with the locations of the vertical stirrup legs (also known as *hanger reinforcement*), which serves as the transverse reinforcement in the stem of the bent cap beam. In the same way, Tie C_sF_s is located at the top horizontal leg of the stirrups provided in the beam ledges. The position of Strut G_sH_s coincides with the location of the bottom chord of the global strut-and-tie model.

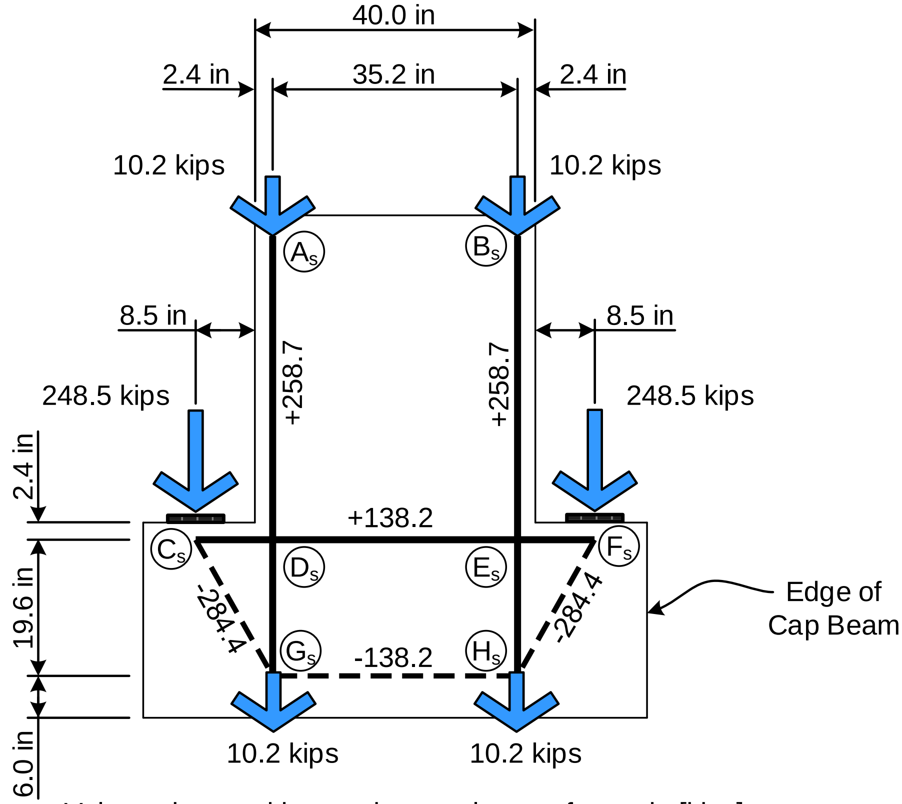


Figure 3-11: Local Strut-and-Tie Model at Beam Line 1

Recall that the reason behind developing the local strut-and-tie models is to design the ledge reinforcement. It is important to note that the development of the local strut-and-tie models is dependent only the applied the beam loads at the beam line in question; the forces in the global strut-and-tie model have no influence on the design of the ledge reinforcement. The area of reinforcement required for Tie C_sF_s in Figure 3-11 is dependent only on the applied beam self-weights and the applied beam reactions.

Inverted-Tee Beam Terminology:

Additional terminology is introduced here to completely describe the reinforcement used in the design of an inverted-tee beam. Refer to Figure 3-12 on the next page.

Hanger reinforcement (or *hanger ties*) refers to the vertical reinforcement in the beam stem within a specified distance from an applied ledge load. The hanger reinforcement carries the beam load upward toward the compression face of the beam.

Ledge reinforcement is the horizontal reinforcement provided in the beam ledges which carries the tensile forces created by the applied ledge loads.

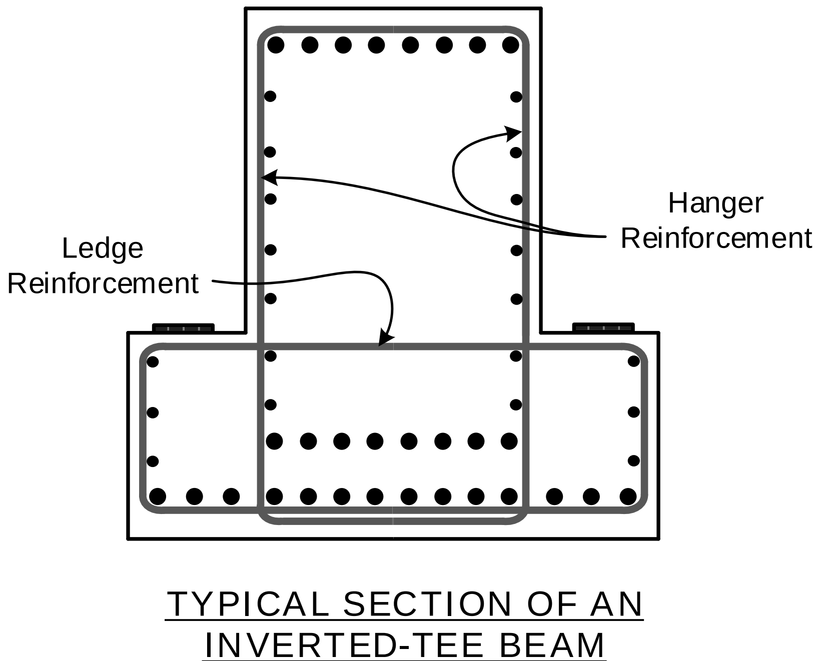
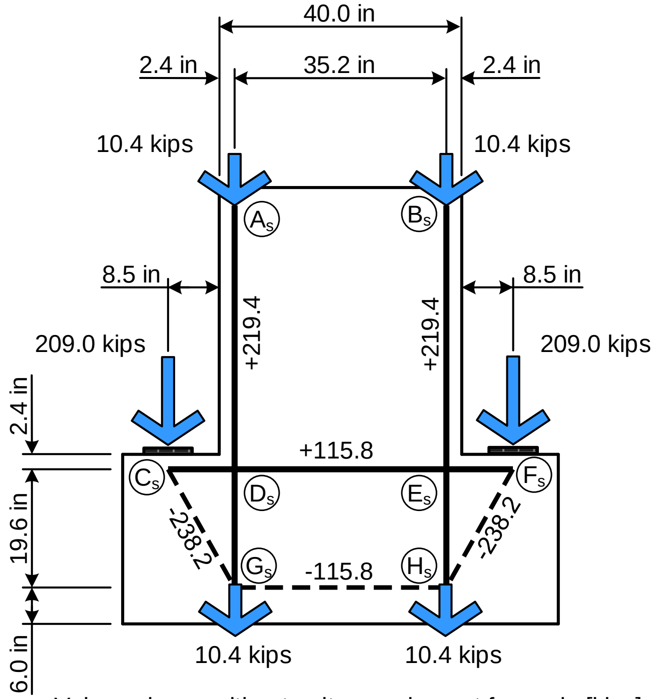


Figure 3-12: Inverted-Tee Beam Terminology

Before continuing, it is worth stating that the designer should keep in mind that the flow of forces within an inverted-tee beam may be visualized as a single, three-dimensional strut-and-tie model. Such a visualization can make it easier to determine if the chords of the truss members are placed correctly. It is therefore reasonable to place Strut G_sH_s such that it would intersect the bottom chord of the global strut-and-tie model.

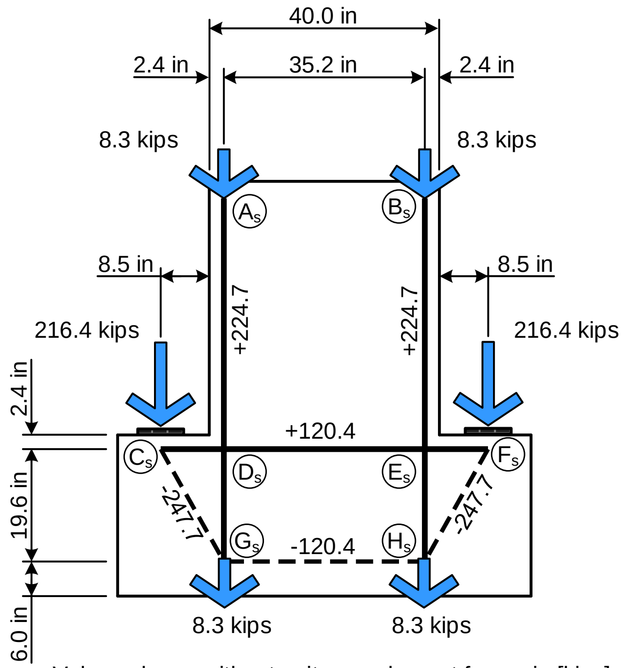
The applied loads in the local strut-and-tie model of Figure 3-11 are the applied factored beam loads (248.5 kips each) and the tributary self-weight of the cap beam, which is evenly distributed to Nodes A_s, B_s, G_s, and H_s (recall that these self-weight loads are factored loads). The individual member forces are then found by satisfying equilibrium at each node.

Local strut-and-tie models must also be developed at Beam Lines 2 and 3. These models are given in Figure 3-13 and Figure 3-14. Each local strut-and-tie model is geometrically identical but is subject to a different set of external forces. By comparing each of the three local models, design of the ledge reinforcement (Tie C_sF_s) and the nodal strength checks will be governed by the model at Beam Line 1 (the location of the largest applied beam loads). In order to simplify detailing and construction, the spacing of the ledge reinforcement required at Beam Line 1 will be provided along the entire ledge. All other reinforcement details will be based on the global strut-and-tie model.



Values shown without units are element forces in [kips]
(+) indicates tension
(-) indicates compression

Figure 3-13: Local Strut-and-Tie Model at Beam Line 2



Values shown without units are element forces in [kips]
(+) indicates tension
(-) indicates compression

Figure 3-14: Local Strut-and-Tie Model at Beam Line 3

Design Step 7 - Proportion Ties

The forces from the global strut-and-tie model will be used to determine the longitudinal reinforcement in the top and bottom chords of the cap beam as well as for the exterior faces of the columns. A constant amount of longitudinal steel will be provided along the cap beam for ease of detailing and construction.

Bottom Chord:

The force in Ties *HI* and *IJ* controls the design of the bottom chord of the global strut-and-tie model. The amount of reinforcing required is found by applying *AASHTO LRFD* Equations 5.8.2.3-1 and 5.8.2.4.1-1:

$$P_u \leq \phi F_y A_{st}$$

$$P_u = 1{,}063.9\ kips$$

$$1{,}063.9\ kips \leq 0.9 \times 60\ ksi \times A_{st}$$

$$A_{st} \geq 19.70\ in^2$$

Using No. 11 reinforcing bars:

$$No.of\ bars = \frac{19.70\ in^2}{1.56\ \frac{in^2}{bar}} = 12.6\ bars$$

Therefore, use 13 No. 11 bars.

Top Chord:

The force in tie *AB* controls the design of the top chord of the global strut-and-tie model. The reinforcing required is determined as for the bottom chord:

$$P_u = 464.3\ kips$$

$$464.3\ kips \leq 0.9 \times 60\ ksi \times A_{st}$$

$$A_{st} \geq 8.60\ in^2$$

Using No. 11 reinforcing bars:

$$No.of\ bars = \frac{8.60\ in^2}{1.56\ \frac{in^2}{bar}} = 5.5\ bars$$

Therefore, use 6 No. 11 bars. Note that this is the minimum amount of reinforcing required. Additional reinforcing may be required to supplement the strength of the nodes. This will be explored in Design Step 9.

Column Vertical Ties:

For simplicity of detailing and construction, identical reinforcing will be provided in both columns. The amount of reinforcing required will be controlled by column tie *AA'*.

$$P_u = 653.9\ kips$$

$$653.9\ kips \leq 0.9 \times 60\ ksi \times A_{st}$$

$$A_{st} \geq 12.11\ in^2$$

Using No. 11 reinforcing bars:

$$No.of\ bars = \frac{12.11\ in^2}{1.56\ \frac{in^2}{bar}} = 7.8\ bars$$

Use 8 No. 11 bars in each column.

Recall that this reinforcing is determined only for one load case. Final reinforcement details should be determined by a complete design that considers all governing load cases.

Hanger Reinforcement (Vertical Ties):

The geometries of the nodes above Beam Lines 1, 2, and 3 are dependent on the distribution of the vertical tie reinforcing at Nodes *C*, *D*, and *E*, respectively.

As opposed to a strut-and-tie model with loads applied to its top chord, a strut-and-tie model loaded by its bottom chord requires hanger reinforcement to transfer the applied superstructure loads to its compression chord. Referring to *AASHTO LRFD* Figure 5.8.4.3.5-2 (reproduced on the next page as Figure 3-15), the length over which the hanger reinforcement may be distributed (i.e., the width of hanger tie) is $(W + 2d_f)$

where:

W =	width of the bearing pad measured along the length of the cap beam, in
d_f =	distance from the top face of the ledge to the centroid of the bottom horizontal leg of the ledge stirrups, in

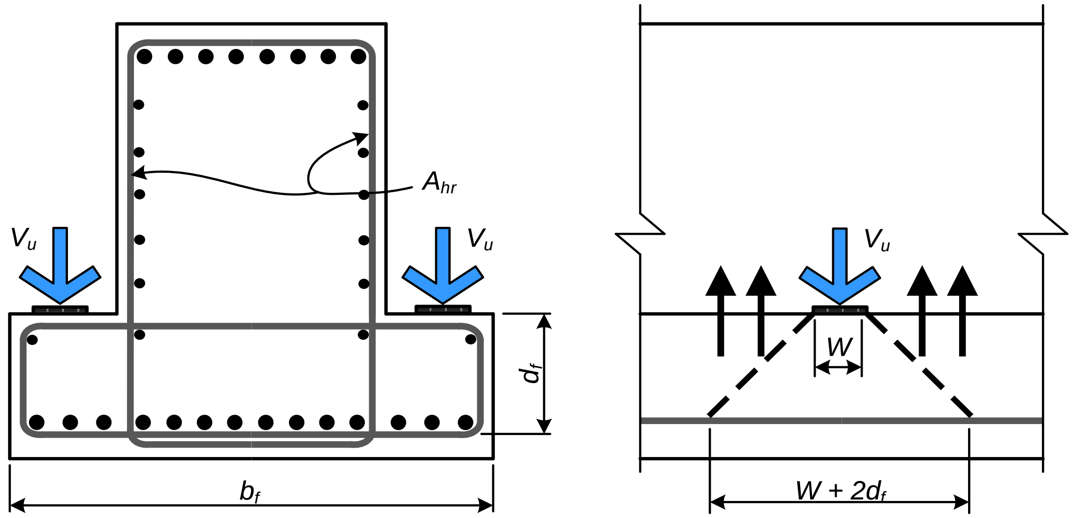


Figure 3-15: Inverted-Tee Beam Hanger Reinforcement

In addition, the effective tie widths for the ledge reinforcement are limited per *AASHTO LRFD* Article 5.8.4.3.5. The distributed width for the interior beam is taken as $(W + 2d_f)$ for interior beams and $(2c)$ for exterior beams, where:

c = distance from the centerline of bearing to end of the beam ledge, in

Any effects of the tapered ends of the beam ledges are conservatively neglected.

Hanger and Ledge Reinforcement:

AASHTO LRFD Article 5.8.4.3.5 covers empirical design of hanger and ledge reinforcing. The empirical guidelines given in this article are followed in this example, even though they are not specifically referenced by the *AASHTO LRFD* strut-and-tie provisions.

The available widths for Ties *CI* and *EK* are determined first. Referring to Figure 3-16 on the next page, the available width is determined thus:

$$(2c)_{CI/EK} = 2 \times 2.58\ ft = 5.17\ ft = 62.0\ in$$

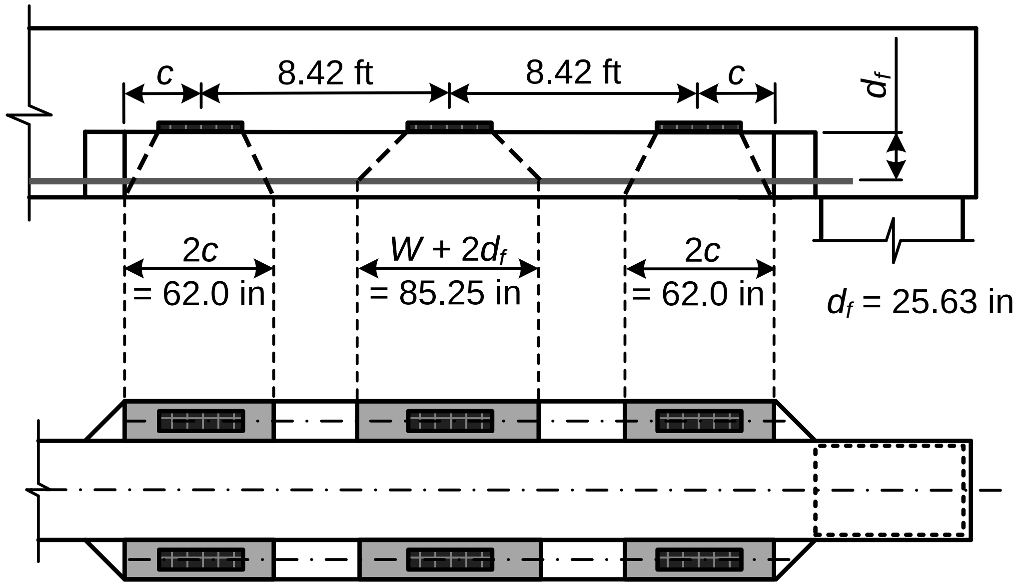


Figure 3-16: Available Hanger Reinforcement Widths

The distance d_f is determined by assuming a 2.0 in clear cover and a No. 6 stirrup, giving:

$$d_f = 28\ in - 2\ in - \frac{0.75\ in}{2} = 25.63\ in$$

Now, the available width of Tie *DJ* is found:

$$W + 2d_f = 34.0\ in + 2 \times 25.63\ in = 85.25\ in$$

The hanger reinforcement along the ledge will be determined first, then the required stirrup spacing for Tie *BH* will be determined.

Tie* EK*:

Tie *EK* is the most critical hanger tie in the bent cap beam because it must carry the largest tensile load with a relatively narrow band of reinforcement. Because of this limitation, bundled No. 6 stirrups with two legs will be used. Alternatively, the designer has the option to use four-legged No. 6 stirrups instead. The required spacing of the stirrups is found in the same manner as for the longitudinal beam ties above, using *AASHTO LRFD* Equations 5.8.2.3-2 and 5.8.2.4.1-1:

$$P_u = 930.5\ kips$$

$$930.5\ kips \leq 0.9 \times 60\ ksi \times A_{st}$$

$$A_{st} \geq 17.23\ in^2$$

Using the No. 6 bundled stirrups:

$$No.of\ stirrups = \frac{17.23\ in^2}{4\ legs \times 0.44\ \frac{in^2}{leg}} = 9.8\ stirrups$$

Recalling that the available width for the stirrups is 62.0 in, the required stirrup spacing is:

$$s \leq \frac{l_a}{No.of\ stirrups} = \frac{62.0\ in}{9.8\ stirrups} = 6.3\ in$$

Thus, use 2 bundled No. 6 stirrups (4 legs total) with a spacing of less than 6.3 in.

Ties* CI *and* DJ*:

Ties *CI* and *DJ* are proportioned next. The reinforcement detailed for Tie *CI* will be used along the entire length of the beam ledge, except at the Tie *EK* region detailed previously. Here, No. 6 stirrups with 2 legs will be used. The required reinforcing is:

$$P_u = 517.4\ kips$$

$$517.4\ kips \leq 0.9 \times 60\ ksi \times A_{st}$$

$$A_{st} \geq 9.58\ in^2$$

The available length is again 62.0 in, giving a required stirrup spacing of:

$$No.of\ stirrups = \frac{9.58\ in^2}{2\ legs \times 0.44\ \frac{in^2}{leg}} = 10.9\ stirrups$$

$$s \leq \frac{l_a}{No.of\ stirrups} = \frac{62.0\ in}{10.9\ stirrups} = 5.7\ in$$

Thus, use No. 6 stirrups (2 legs total) with a spacing of less than 5.7 in.

Tie* BH*:

In contrast to Nodes *C*, *D*, and *E*, Nodes *B* and *H* are smeared nodes with undefined geometries. Hence, Tie *BH* is contained within a fan-shaped strut which connects Nodes *C* and *G*, and the reinforcement for Tie *BH* will be determined using the method explained in Design Example 1, Design Step 7. The shear span, *a*, is the distance between Nodes *C* and *G*:

$$a = 8.62\ ft \times 2 = 17.24\ ft \times \frac{12\ in}{ft} = 206.9\ in$$

Referring back to Figure 3-10, the height of the global strut-and-tie model truss is 5.12 ft, or 61.4 in. The available width for the tie is then:

$$l_a = a - 2 \times (h_{stm} \times \tan 25°)$$

$$l_a = 206.9\ in - 2 \times 61.4\ in \times \tan 25° = 149.6\ in$$

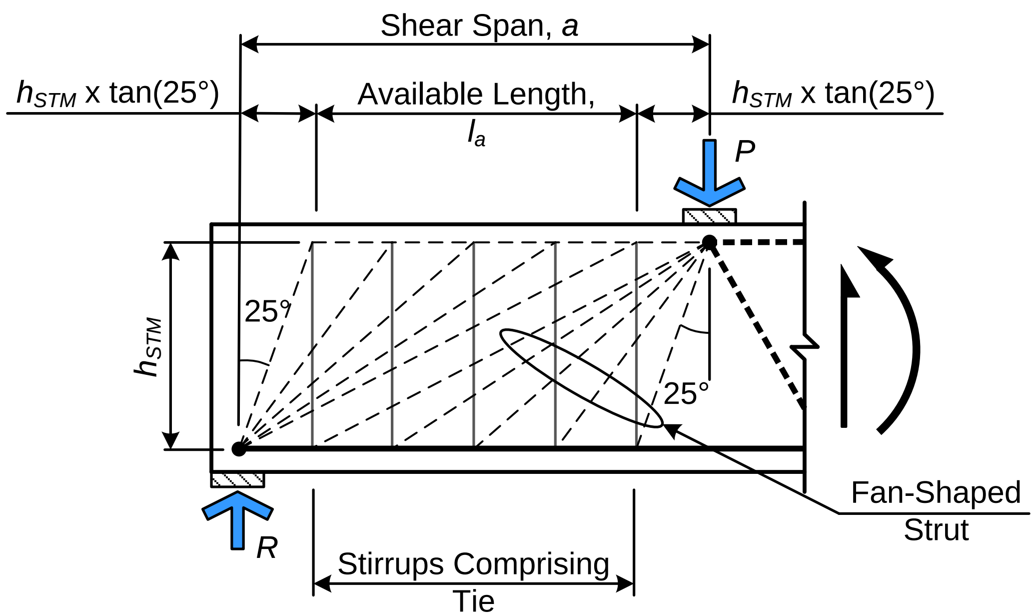


Figure 3-17: Geometry of a Fan-Shaped Strut (Excerpt of *AASHTO LRFD* Figure C5.8.2.2-2)

However, in reality, this available length l_a will be partially occupied by the reinforcement for Tie *CI*. Therefore, the reinforcement must be spread over a smaller distance, chosen to be equal to the average spacing between Nodes *G*, *H*, and *I*, which is 8.62 ft, or 103.5 in. Using No. 6 stirrups with 2 legs, the stirrups will be centered on the tie and spaced evenly over the available length. Applying *AASHTO LRFD* Equations 5.8.2.3-1 and 5.8.2.4.1-1:

$$P_u = 532.3\ kips$$

$$532.3\ kips \leq 0.9 \times 60\ ksi \times A_{st}$$

$$A_{st} \geq 9.86\ in^2$$

$$No. of\ stirrups = \frac{9.86\ in^2}{2\ legs \times 0.44\ \frac{in^2}{leg}} = 11.2\ stirrups$$

$$s \leq \frac{l_a}{No. of\ stirrups} = \frac{103.5\ in}{11.2\ stirrups} = 9.2\ in$$

Thus, use No. 6 stirrups (2 legs total) with a spacing of less than 9.2 in.

It will be demonstrated that the minimum crack control reinforcement required will ultimately control the detailing in this region of bent cap beam. Crack control reinforcement is detailed in Design Step 10.

Ties* A'G' *and* L'F'*:

Ties *A'G'* and *L'F'* distribute the shear that results from the moment frame analysis into the strut-and-tie model of the cap beam. These ties are required because the presence of these ties influences the forces in the global strut-and-tie model. These ties are also anchored by smeared nodes, therefore they will be proportioned in the same manner as Tie *BH*.

The available shear span, *a*, is taken as the distance between Nodes *G* and *G'*, or 60.0 in. The "height" of the STM in these locations will be taken as the smaller of the distances between Nodes *A'* and *G'* and Nodes *L'* and *F'*. This is found to be 3.54 ft, or 42.5 in, between Nodes *L'* and *F'*. Thus:

$$l_a = a - 2 \times (h_{stm} \times \tan 25°)$$

$$l_a = 60.0\ in - 2 \times (42.5\ in \times \tan 25°) = 20.4\ in$$

This reinforcement will be provided on either side of Ties *A'G'* and *L'F'* as No. 6 closed column ties. The column ties will be centered on the each tie and spaced evenly over the available length. Applying *AASHTO LRFD* Equations 5.8.2.3-1 and 5.8.2.4.1-1:

$$P_u = 268.9\ kips$$

$$268.9\ kips \leq 0.9 \times 60\ ksi \times A_{st}$$

$$A_{st} \geq 4.98\ in^2$$

$$No.of\ stirrups = \frac{4.98\ in^2}{2\ legs \times 0.44\ \frac{in^2}{leg}} = 5.7\ stirrups$$

$$s \leq \frac{l_a}{No.of\ stirrups} = \frac{20.4\ in}{5.7\ stirrups} = 3.6\ in$$

Thus, use No. 6 closed column ties (2 legs total) with a spacing of less than 3.6 in.

Design Step 8 - Proportion Ledge Reinforcement

As was stated earlier, Tie C_sF_s of the local strut-and-tie model at Beam Line 1 (refer to Figure 3-11) was found to control the design of the ledge reinforcement. According to *AASHTO LRFD* Article 5.8.4.3.3, the reinforcement encompassing this tie should be uniformly spaced over a distance of $(W + 5a_f)$ or $(2c)$, whichever is less, where:

a_f = distance from centerline of girder reaction to vertical reinforcement in backwall or stem of inverted tee, in

subject to the limitation that the widths of these regions shall not overlap. The distribution of this reinforcement is given in *AASHTO LRFD* Figure 5.8.4.3.3-1, reproduced below as Figure 3-18. The limit of $(W + 5a_f)$ may be applied to the interior beam, and the limit of $(2c)$ may be applied for the exterior beams.

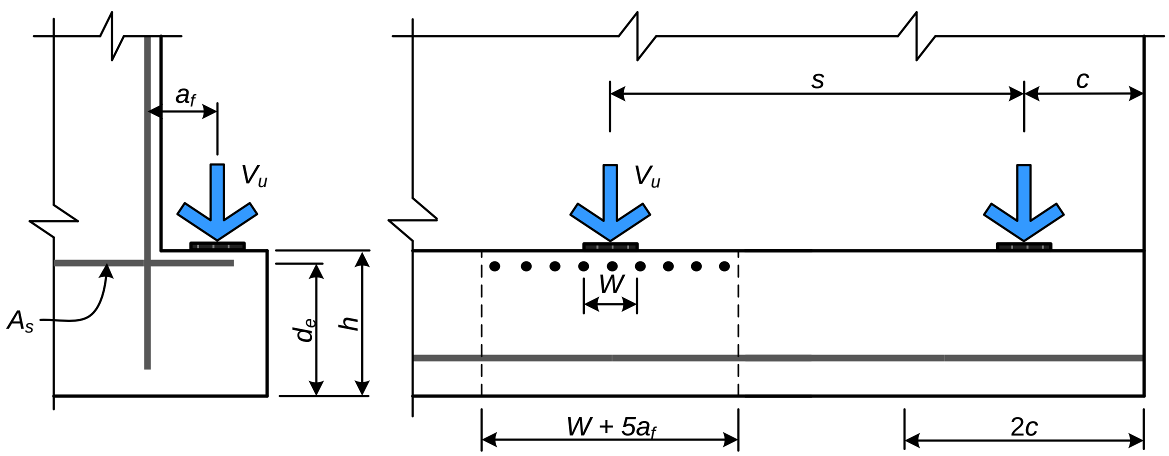


Figure 3-18: Flexural Reinforcement for Ledges (*AASHTO LRFD* Figure 5.8.4.3.3-1)

Three-Dimensional Judgement:

Consider again the three-dimensional flow of forces within the inverted-tee bent cap beam. The ledge reinforcement and hanger reinforcement must work together to carry the applied beam forces around the cross section of the cap beam. In this design example, recall that the available width of Tie *CI* is 62.0 in. The job of Tie *CI* is to “hang up” the ledge reinforcement, which is represented by Tie C_SF_S in the local STM at Beam Line 1. Therefore, instead of applying the provisions of AASHTO LRFD Article 5.8.4.3.3, the width of Tie C_SF_S will be limited to the width of Tie *CI*, or 62.0 in.

In this design example, the width of Tie *CI* just so happens to match the result obtained by applying the $(2c)$ limitation for an exterior beam:

$$2c = 2 \times 2.58\,ft \times \frac{12\,in}{ft} = 62.0\,in$$

Since the force in Tie C_SF_S of the local strut-and-tie model at Beam Line 1 controls, and the available width of Tie C_SF_S is smaller at the exterior beams than the interior beam, using the spacing determined using the forces and available width at Tie C_SF_S to determine the required reinforcement will result in a conservative design for the entire beam ledge. Assuming No. 6 reinforcing bars will be used for the ledge and applying *AASHTO LRFD* Equations 5.8.2.3-1 and 5.8.2.4.1-1:

$$P_u = 138.2\ kips$$

$$138.2\ kips \leq 0.9 \times 60\ ksi \times A_{st}$$

$$A_{st} \geq 2.56\ in^2$$

$$No.of\ bars = \frac{2.56\ in^2}{0.44\ \frac{in^2}{bar}} = 5.8\ bars$$

$$s \leq \frac{l_a}{No.of\ bars} = \frac{62.0\ in}{5.8\ bars} = 10.7\ in$$

Thus, use No. 6 bars with a spacing of less than 10.7 in.

Providing stirrups within the ledge satisfies this requirement (see Figure 3-19 on the following page). To simplify construction, each of the ledge stirrups will be paired with the stirrups in the cap beam stem. Since the required stirrup spacings for the cap beam are all less than that required for the ledge reinforcing (i.e., they are all less than 10.7 in), pairing the stirrups this way ensures sufficient reinforcement is provided over the entire ledge length.

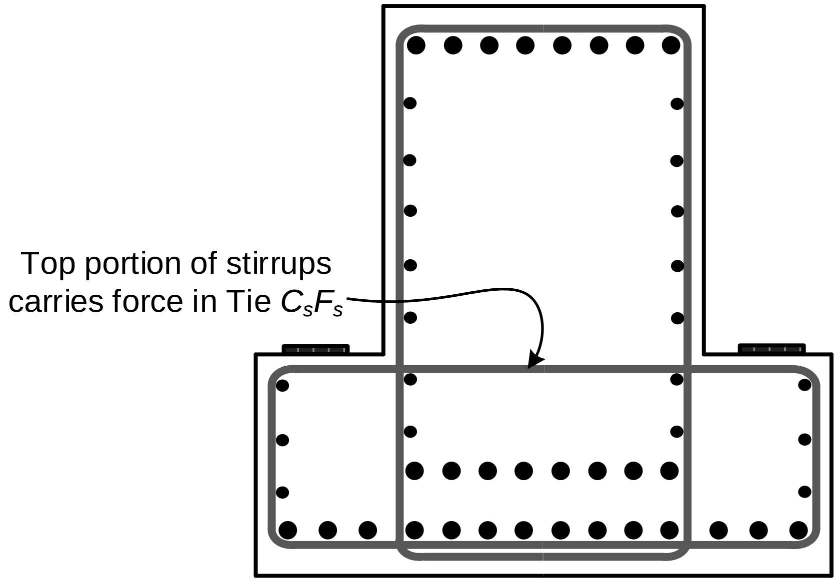


Figure 3-19: Stirrups Carrying Tie Force C_sF_s

Design Step 9 - Perform Nodal Strength Checks

Figure 3-20 is a representation of how the struts and nodes fit within the global strut-and-tie model of the inverted-tee beam. An arbitrary size is given to smeared Nodes *B* and *H*, as they are only drawn for illustrative purposes. The nodes with multiple intersecting struts may be resolved to simplify the nodal geometries.

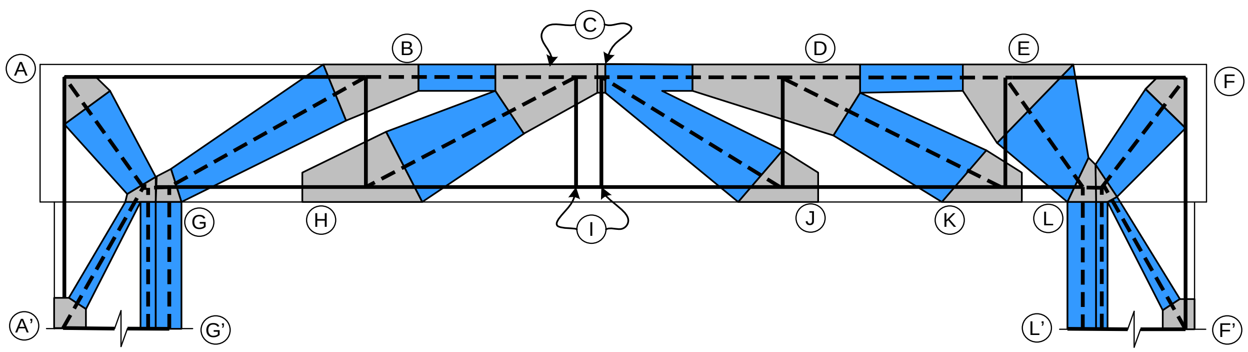


Figure 3-20: Struts and Nodes within the Inverted-Tee Cap Beam

Within Design Step 9, the nodes of the global strut-and-tie model will be evaluated first. The most critical nodes will be identified and the corresponding strength checks will be carried out. Some of the remaining nodes may be deemed to have adequate strength by inspection. Nodes *A* and *F* are "curved bar" nodes, which will be explained later in this design step. The nodes in the local strut-and-tie model at Beam Line 1 will then be evaluated.

Check Node G *(CCC/CCT):*

Nodes *G* and *L* are located near the inside faces of the frame corners. Due to the tight geometric constraints and large forces acting on the nodes, these nodes are among the most highly-stressed regions in the bent cap beam. The geometry of Node *G* is given in Figure 3-21. The total width of the bearing (bottom) face of the node is taken as twice the distance from the face of Column A to the centerline of Strut *GG'*, 2.10 ft or 25.2 in. The height of the back face of the node is taken as twice the distance from the bottom surface of the bent cap beam to the centroid of the bottom chord reinforcement, or 12.0 in. Because multiple struts intersect at Node *G*, the node will be subdivided and struts will be resolved to ensure that no more than three forces intersect at a single node.

The two diagonal Struts *AG* and *A'G* will be resolved into a single strut called Strut *AA'G*. The force acting on the bearing face of the left portion of the node equilibrates the vertical component of the strut acting on the left node face (Strut *AA'G*) and a portion of the applied self-weight. Equilibrium is satisfied for the right portion of the node in the same manner. Note that the inclinations of the struts must also be revised to account for the subdivision of the node. The new inclination angles are shown with the original inclination angles in Figure 3-21:

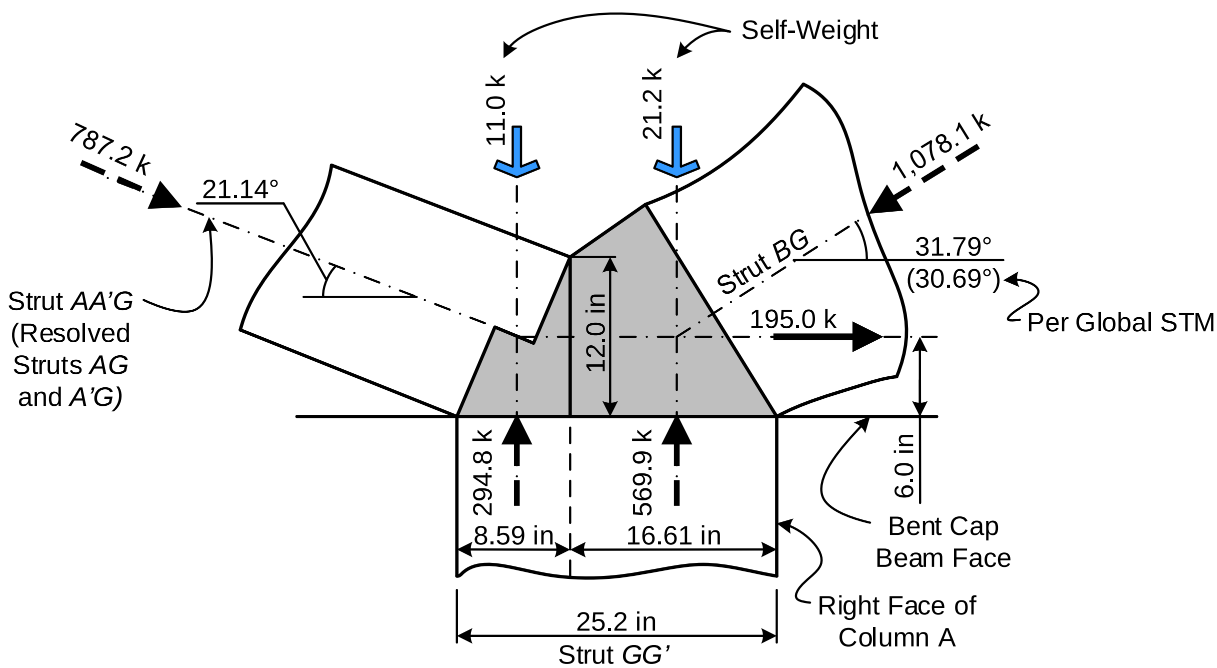


Figure 3-21: Geometry of Node *G*

The dimension of the bearing face for each nodal subdivision is determined based on the magnitude of the vertical component of each diagonal strut in relation to the net vertical force from Strut *GG'* and the applied self-weight at the node. Uniform bearing pressure will be maintained over the total 25.2 in width of Strut *GG'* to maintain equilibrium.

First, resolve Struts *AG* and *A’G* into a single strut, Strut *AA’G*:

$$\sum_{+\uparrow} F_{Vert} = -802.0\ kips \times \left(\frac{5.12\ ft}{6.28\ ft}\right) + 457.3\ kips \times \left(\frac{5.00\ ft}{6.18\ ft}\right) = -283.9\ kips$$

$$\sum_{+\rightarrow} F_{Horiz} = 802.0\ kips \times \left(\frac{3.64\ ft}{6.28\ ft}\right) + 457.3\ kips \times \left(\frac{3.64\ ft}{6.18\ ft}\right) = 734.2\ kips$$

$$F_{AA'G} = \sqrt{(-283.9\ kips)^2 + (734.2\ kips)^2} = 787.2\ kips$$

The angle of inclination of the resolved strut is found next:

$$\theta_{AA'G} = \tan^{-1}\left[\frac{Vertical\ Force}{Horizontal\ Force}\right] = \tan^{-1}\left[\frac{283.9\ kips}{734.2\ kips}\right] = 21.14°$$

The length of each bearing face is determined thus:

$$width_{left} = \left[\frac{787.2\ kips \times \sin 21.14°}{864.7\ kips - 32.2\ kips}\right] \times 25.2\ in = 8.59\ in$$

$$width_{right} = 25.2\ in - 8.59\ in = 16.61\ in$$

where:

- 864.7 kips is the force in Strut *GG’*
- 32.2 kips is the self-weight applied at Node *G*
- 787.2 kips is the force in Strut *AA’G*
- 21.14° is the angle of inclination of Strut *AA’G*

The revised angle of inclination of Strut *BG* is now calculated:

$$\theta_{BG} = \tan^{-1}\left[\frac{61.44\ in}{103.44\ in - \left(\frac{25.2\ in}{2} - \frac{16.61\ in}{2}\right)}\right] = 31.79°$$

where:

- 61.44 in is the height of the strut-and-tie model (h_{stm})
- 44.16 in is the horizontal distance from Node *G* to Tie *AA’*
- 103.44 in is the horizontal distance from Node *G* to Node *H*
- 16.61 in is the subdivided bearing face width for Strut *BG*

The widths of the strut-to-node interfaces, w_s, are now calculated:

$$w_{s,left} = l_b \sin\theta + a\cos\theta$$

$$w_{s,left} = 8.59\ in \times \sin 21.14° + 12.0\ in \times \cos 21.14° = 14.29\ in$$

$$w_{s,right} = l_b \sin\theta + a\cos\theta$$

$$w_{s,right} = 16.61\ in \times \sin 31.79° + 12.0\ in \times \cos 31.79° = 18.95\ in$$

Only compression forces act on the left portion of Node *G*, while one tensile force acts on the right half of Node *G*. Therefore, the left portion will be treated as a CCC node, and the right half will be treated as a CCT node.

Node* G_{Right} *(CCT):

Because the bent cap beam is wider than both columns, the confinement modification factor, *m*, may be applied to the strength of Node G_{Right}. The value of *m* is determined by applying *AASHTO LRFD* Equation 5.6.5-3, where the values of A_1 and A_2 are determined using Figure 3-22:

$$m = \sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{40.0\ in \times 29.2\ in}{25.2\ in \times 36.0\ in}} = 1.13 < 2$$

Since the calculated value of *m* is less than 2, use the calculated value of 1.13.

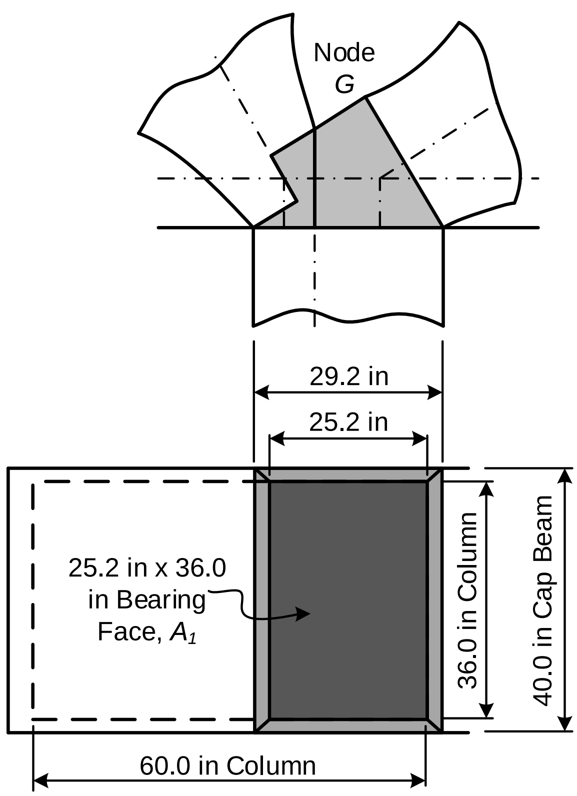


Figure 3-22: Confinement Modification Factor, *m*, at Node *G*

Applying the Confinement Modification Factor:

Experimental results by Bayrak et al. and evaluation of existing deep beam tests suggest that the benefits of the triaxial confinement effect are applicable to all faces of a node due to the confining effect of the concrete surrounding the node. In this design example, it is deemed appropriate to apply the confinement modification factor to all of the faces of Node G_{Right} because:

- the adjoining Node G_{Left} provides restraint to the back face of the node
- the column provides restraint to the bearing face of the node

Please note that not taking advantage of the triaxial confinement effect would result in a more conservative design by reducing the allowable concrete stresses at the node faces. It is up to the designer to determine if using the confinement modification factor is appropriate. Please see the references for additional information.

Each face of the subdivided node is now checked using the nodal strength checks of *AASHTO LRFD* Article 5.8.2.5. Begin by applying *AASHTO LRFD* Equations 5.8.2.3-1 and 5.8.2.5.1-1:

$$\phi P_n = \phi m v f_c' A_{cn}$$

The concrete efficiency factors, *v*, may be found in *AASHTO LRFD* Table 5.8.2.5.3a-1.

For the bearing face:

$$P_u = 569.9\ kips$$

$$f_{cu} = mvf'_c = 1.13 \times 0.7 \times 6.0\ ksi = 4.75\ ksi$$

$$\phi P_n = 0.7 \times 4.75\ ksi \times 36.0\ in \times 16.61\ in = 1{,}988.2\ kips$$

$$1{,}988.2\ kips > 569.9\ kips \quad \boldsymbol{OK}$$

For the back face:

$$P_u = 787.2\ kips \times \cos 21.14° = 734.2\ kips$$

$$f_{cu} = mvf'_c = 1.13 \times 0.7 \times 6.0\ ksi = 4.75\ ksi$$

$$\phi P_n = 0.7 \times 4.75\ ksi \times 40.0\ in \times 12.0\ in = 1{,}596.0\ kips$$

$$1{,}596.0\ kips > 734.2\ kips \quad \boldsymbol{OK}$$

For the strut-to-node interface:

$$P_u = 1{,}078.1\ kips$$

$$v = 0.85 - \frac{f'_c}{20.0\ ksi} \quad (0.45 \leq v \leq 0.65)$$

$$v = 0.85 - \frac{6.0\ ksi}{20.0\ ksi} = 0.55$$

$$f_{cu} = mvf'_c = 1.13 \times 0.55 \times 6.0\ ksi = 3.73\ ksi$$

$$\phi P_n = 0.7 \times 3.73\ ksi \times 40.0\ in \times 18.95\ in = 1{,}979.1\ kips$$

$$1{,}979.1\ kips > 1{,}078.1\ kips \quad \boldsymbol{OK}$$

Node* G$_{Left}$ *(CCC):

The pressures acting on the back face of the left portion of Node *G* are the same as the right portion due to equilibrium. The pressure acting on the bearing face of Node G_{Left} is the same as the right portion (recall that Strut *GG'* was divided by maintaining equal pressures in each portion of the subdivided strut). Therefore, only the strut-to-node interface of Node G_{Left} need be checked:

For the strut-to-node interface:

$$P_u = 787.2\ kips$$

$$v = 0.85 - \frac{f'_c}{20.0\ ksi} \quad (0.45 \leq v \leq 0.65)$$

$$v = 0.85 - \frac{6.0\ ksi}{20.0\ ksi} = 0.55$$

$$f_{cu} = mvf'_c = 1.13 \times 0.55 \times 6.0\ ksi = 3.73\ ksi$$

$$\phi P_n = 0.7 \times 3.73\ ksi \times 40.0\ in \times 14.29\ in = 1{,}492.5\ kips$$

$$1{,}492.5\ kips > 787.2\ kips \quad \boldsymbol{OK}$$

Hence, Node *G* is sufficient to resist the applied forces.

Node* L *(CCC/CCT):

For Node *L*, the geometry and subdivision of forces is carried out in exactly the same way as for Node *G*. Calculations show that all of the nodal faces at Node *L* have adequate strength to resist the applied forces.

Node C *(CCT):*

The nodal strength checks for Node *C* are performed next. The diagonal Strut *CH* entering the node is highly stressed and large compressive forces act on a relatively small area on the back face of Node *C.* This node is therefore a critical node to check. Because struts enter the node on its left and right faces, this node is subdivided into two parts (see Figure 3-23). The total length of the top nodal face is assumed to be the same as the width of the corresponding hanger tie (Tie *CI*). The width of the top face is consequently taken as 5.17 ft, or 62.0 in. The height of the back face is taken as double the distance from the top of the bent cap beam to the centroid of the top chord reinforcement (9.2 in).

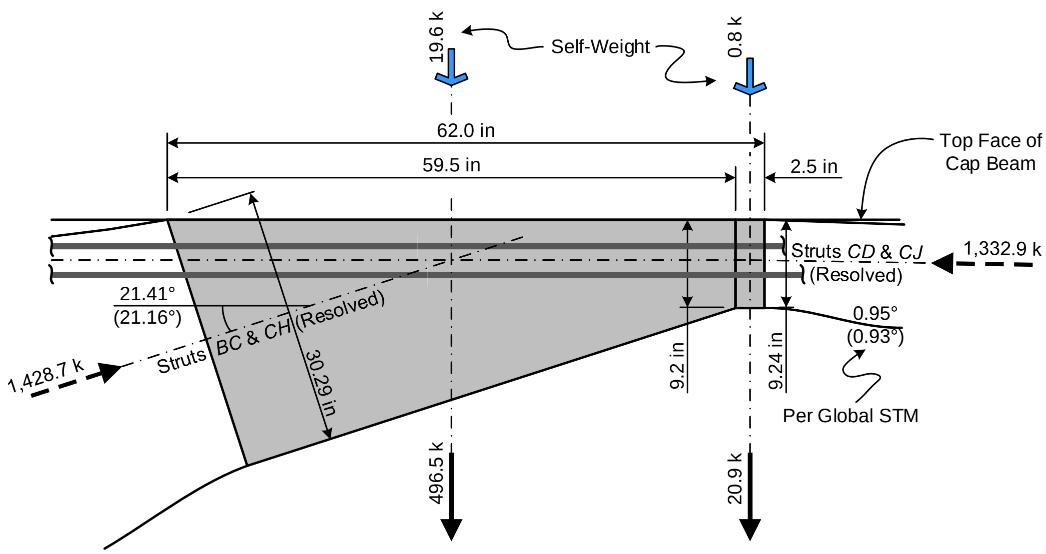


Figure 3-23: Geometry of Node *C*

Here, the length of the top face for each nodal subdivision is based on the magnitude of the vertical component of each diagonal strut in relation to the net vertical force from Tie *CI* and the applied self-weight at Node *C* (this approach is exactly the same as was done for Node *G*). The length of each nodal top face is:

$$Left = \left[\frac{1{,}010.5\ kips \times \sin 30.71°}{517.4\ kips + 20.4\ kips}\right] \times 62.0\ in = 59.5\ in$$

$$Right = 62.0\ in - 59.5\ in = 2.5\ in$$

where:

- 1,010.5 kips is the force in Strut *CH*
- 20.4 kips is the self-weight applied at Node *C*
- 517.4 kips is the force in Tie *CI*
- 30.71° is the angle of inclination of Strut *CH*

Note the difference in size between the left and right portion of Node *C*. Before revising the diagonal strut angles to reflect the subdivided node, the struts adjacent to Node *C* are resolved to reduce the number of forces acting on the node. Struts *BC* and *CH* and Struts *CD* and *CJ* are resolved into two struts acting on the left and right faces of Node *C*, respectively. The resolved geometry is given in Figure 3-23.

The resolved strut acting on the left face of Node *C* is determined thus:

$$F_{Left,res} = \sqrt{(463.9\ kips + 868.4\ kips)^2 + (515.8\ kips + 0\ kips)^2}$$

$$F_{Left,res} = 1{,}428.7\ kips$$

where:

- 463.9 kips is the horizontal component of the force in Strut *BC*
- 868.4 kips is the horizontal component of the force in Strut *CH*
- 515.8 kips is the vertical component of the force in Strut *CH*
- 0 kips is the vertical component of the force in Strut *BC*

The angle of inclination of the resolved strut is:

$$\theta_{Left,res} = \tan^{-1}\left(\frac{Vertical\ Force}{Horizontal\ Force}\right)$$

$$\theta_{Left,res} = \tan^{-1}\left(\frac{515.8\ kips + 0\ kips}{463.9\ kips + 868.4\ kips}\right) = 21.16°$$

Similarly, the resolved strut force and inclination angle are determined for the right face of Node *C*:

$$F_{Right,res} = 1{,}332.9\ kips$$

$$\theta_{Right,res} = 0.93°$$

Subsequently, the resolved strut inclination angles are revised to reflect the subdivided nodal geometry. The angle of inclination for the resolved strut on the left of Node *C* is calculated thus:

$$\tan(21.16°) = \frac{61.44\ in}{x\ in} \rightarrow\ x = 157.96\ in$$

$$\theta_L = \tan^{-1}\left[\frac{61.44\ in}{157.96\ in - \left(\frac{62.0\ in}{2} - \frac{59.5\ in}{2}\right)}\right] = 21.41°$$

and the angle of inclination for the resolved strut on the right of Node *C* is:

$$\tan(0.93°) = \frac{61.44\ in}{x\ in} \rightarrow\ x = 3{,}784.89\ in$$

$$\theta_L = \tan^{-1}\left[\frac{61.44\ in}{3{,}784.89\ in - \left(\frac{62.0\ in}{2} - \frac{2.5\ in}{2}\right)}\right] = 0.94°$$

Finally, the widths of the new strut-to-node interfaces are calculated:

$$w_{s,left} = l_b \sin\theta + a\cos\theta$$

$$w_{s,left} = 59.5\ in \times \sin 21.41° + 9.2\ in \times \cos 21.41° = 30.29\ in$$

$$w_{s,right} = l_b \sin\theta + a\cos\theta$$

$$w_{s,right} = 2.5\ in \times \sin 0.94° + 9.2\ in \times \cos 0.94° = 9.24\ in$$

Node* C_{Left} *(CCT):

Node *C* has no bearing surface, therefore no bearing check is necessary. Longitudinal reinforcement is provided along the entire length of the bent cap beam. So long as this reinforcement is detailed to develop its yield stress in compression, the longitudinal reinforcing will contribute to the strength of the back face of Node *C*. Recall that the top chord reinforcing determined in Design Step 7 was 6 No. 11 reinforcing bars. Using this amount of reinforcing, the back face of the node may be checked using *AASHTO LRFD* Equations 5.8.2.3-1 and 5.8.2.5.1-1, modified by including the strength of the reinforcing steel and subtracting the area of reinforcing steel from the area of concrete. Alternatively, *AASHTO LRFD* Equation 5.6.4.4-3 may be used.

For the back face:

$$m = 1.0$$

$$P_u = 1{,}332.9\ kips$$

$$f_{cu} = mvf'_c = 1.0 \times 0.7 \times 6.0\ ksi = 4.20\ ksi$$

$$\phi P_n = \phi\left(mvf'_c A_{cn,net} + A_s F_y\right)$$

$$A_{cn.net} = A_{cn} - A_s = [9.2\ in \times 40.0\ in] - \left[6\ bars \times 1.56\ \frac{in^2}{bar}\right] = 358.6\ in^2$$

$$\phi P_n = 0.7 \times \left(4.20\ ksi \times 358.6\ in^2 + 6\ bars\ \times 1.56 \frac{in^2}{bar} \times 60\ ksi\right)$$

$$\phi P_n = 1{,}447.4\ kips$$

$$1{,}447.4\ kips > 1{,}332.9\ kips\quad \boldsymbol{OK}$$

For the strut-to-node interface:

$$m = 1.0$$

$$P_u = 1{,}428.7\ kips$$

$$v = 0.85 - \frac{6.0\ ksi}{20.0\ ksi} = 0.55$$

$$f_{cu} = mvf'_c = 1.0 \times 0.55 \times 6.0\ ksi = 3.30\ ksi$$

$$\phi P_n = \ \phi mvf_c' A_{cn}$$

$$\phi P_n = 0.7 \times 3.30\ ksi \times 40.0\ in \times 30.29\ in = 2{,}798.8\ kips$$

$$2{,}798.8\ kips > 1{,}428.7\ kips \quad \boldsymbol{OK}$$

Node* C_{Right} *(CCT):

Checks of the right portion of Node *C* are carried out in the same manner as for the left portion. The check for the back face of the node is identical to the check for the left portion of Node *C*, so it will not be reproduced here.

For the strut-to-node interface:

$$m = 1.0$$

$$P_u = 1{,}332.9\ kips$$

$$v = 0.85 - \frac{6.0\ ksi}{20.0\ ksi} = 0.55$$

$$f_{cu} = mvf'_c = 1.0 \times 0.55 \times 6.0\ ksi = 3.30\ ksi$$

$$\phi P_n = \ \phi\left(mvf_c' A_{cn,net} + A_s F_y\right)$$

$$A_{cn.net} = A_{cn} - A_s = [9.24\ in \times 40.0\ in] - \left[6\ bars \times 1.56\ \frac{in^2}{bar}\right] = 360.24\ in^2$$

$$\phi P_n = 0.7 \times \left(3.30\ ksi \times 360.24\ in^2 + 6\ bars\ \times 1.56\frac{in^2}{bar} \times 60\ ksi\right) = 1{,}225.3\ kips$$

$$1{,}225.3\ kips < 1{,}332.9\ kips \quad \boldsymbol{NO\ GOOD}$$

The strut-to-node interface calculations indicate that the node face does not have the strength required to resist the resolved strut force. However, by examining the resolved strut, its angle of inclination is practically zero, and may essentially be neglected, making the check at this node face the same as the back face check for the left portion of Node *C*. Therefore, ***SAY OKAY.***

Node I *(Ties Only):*

Node *I* is located directly below Beam Line 1. Referring to the global strut-and-tie model in Figure 3-10, only ties intersect at this node. Nodal checks are therefore unnecessary because no compressive force act on the node. However, the strength of the bearings at Beam Line 1 must be checked to ensure adequate strength against bearing failure. These checks should be performed as part of the local strut-and-tie model evaluation.

Node K *(CTT):*

Node *K*, directly below Beam Line 3, is shown in Figure 3-24. The length of the bottom face of the node is conservatively chosen to the width of the bearing pad, *W*. Alternatively, the designer may wish to reduce the nodal stresses by accounting for the lateral distribution of the applied beam load through the depth of the ledge; by considering this distribution, the length of the bottom face of the node would increase. This approach is not necessary to satisfy the nodal strength check for this design example.

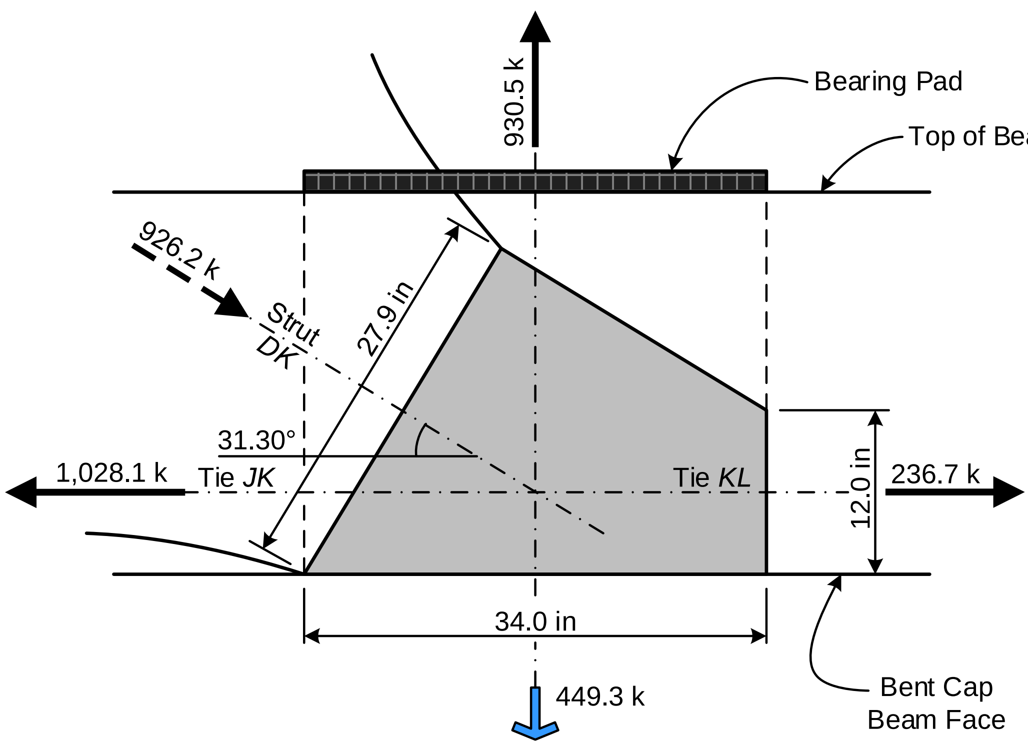


Figure 3-24: Geometry of Node *K*

In spite of the presence of a bearing pad on the ledge, a bearing force does not act directly on the node; hence, the confinement modification factor cannot be applied at Node *K*. In addition, recall that the nodes of the global strut-and-tie model are assumed to be confined within the stem of the inverted-tee beam and not the ledges. Note that a width of 40.0 in is used for the width of the strut-to-node interface design check below. The distance l_b will be restricted to the width of the bearing pad, 34.0 in.

The width of the strut-to-node interface is first determined:

$$w_{s,left} = l_b \sin\theta + a\cos\theta$$

$$w_{s,left} = 34.0\ in \times \sin 31.30° + 12.0\ in \times \cos 31.30° = 27.9\ in$$

For the strut-to-node interface:

$$m = 1.0$$

$$P_u = 926.2\ kips$$

$$v = 0.85 - \frac{6.0\ ksi}{20.0\ ksi} = 0.55$$

$$f_{cu} = mvf'_c = 1.0 \times 0.55 \times 6.0\ ksi = 3.30\ ksi$$

$$\phi P_n = \phi mvf'_c A_{cn}$$

$$\phi P_n = 0.7 \times 3.30\ ksi \times 40.0\ in \times 27.9\ in = 2{,}578.0\ kips$$

$$2{,}578.0\ kips > 926.2\ kips \quad \boldsymbol{OK}$$

The Back Face of the Node:

Recall that *AASHTO LRFD* Article C5.8.2.5.3b states that there is no research that showed bond stress of reinforcement controlling the strength of a nodal region. Therefore, it is not necessary to check the strength of the back face of Node *K*.

Hence, Node *K* is sufficient to resist the applied forces.

Nodes* A *and* F *(CTT):

Nodes *A* and *F* are referred to as "*curved-bar nodes*". Curved-bar nodes are not included in the *AASHTO LRFD* specifications, and because these nodes are not highly stressed, these nodes will not be checked.

Curved-Bar Nodes:

A *curved-bar* node results when a large-diameter reinforcing bar (i.e., a No. 11 or No. 18 bar) is bent around a corner. The geometry of a curved-bar node is shown in Figure 3-25 on the following page. This type of node is not yet included in the *AASHTO LRFD* specifications because it has not yet been vetted by as much experimental data as for CCC, CCT, and CTT nodes. For additional information, refer to TxDOT research report 5-5253-01 (Williams et al., 2011).

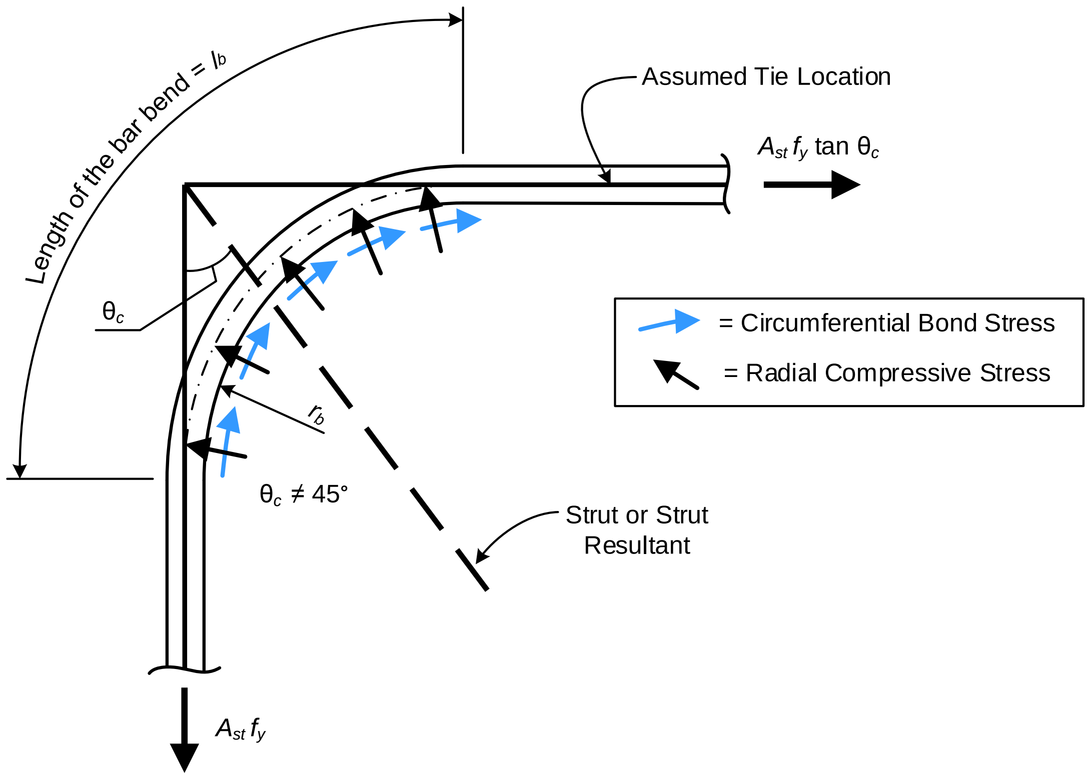


Figure 3-25: Geometry of a Curved-Bar Node

Nodes C_s *and* F_s *(Local STM, CCT):*

Nodes C_s and F_s of the local strut-and-tie model at Beam Line 1 (Figure 3-11) are the most critical nodes of the three local strut-and-tie models. Since the nodes are identical (the local strut-and-tie model is symmetrical about the cap beam centerline), only one needs to be checked. The geometry of Node C_s is given in Figure 3-26 on the following page. The length of the bearing face of the node is taken as the width of the bearing pad, or 8.0 in, and the height of the back face is taken as double the distance from the top surface of the ledge to the top horizontal leg of the ledge stirrup, or 4.8 in. The width of the node in and out of the page is assumed to be the length of the bearing pad, or 34.0 in.

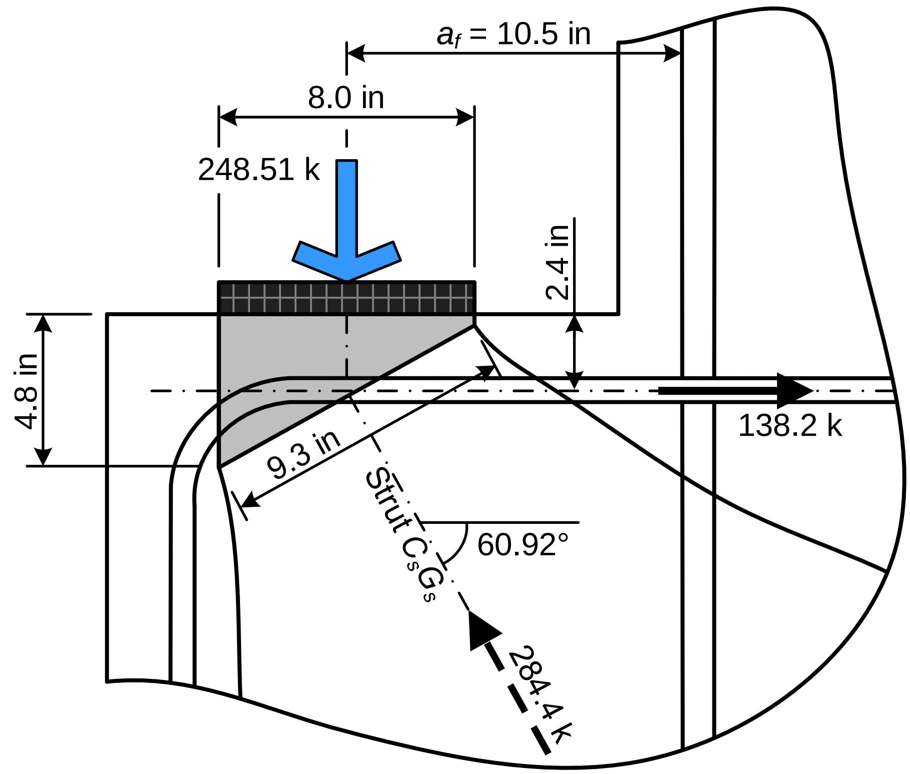


Figure 3-26: Node *Cs* of Local Strut-and-Tie Model at Beam Line 1

To simplify calculation, the confinement modification factor, *m*, is conservatively taken as 1.0. It will be demonstrated that all of the nodal faces have sufficient strength to resist the applied loads without consideration of the effects of triaxial confinement. The demand on the bearing face of the node is equal to the reaction from the trapezoidal box beam. The width of the strut-to-node interface is first determined:

$$w_s = l_b \sin\theta + a\cos\theta$$

$$w_s = 8.0\ in \times \sin 60.92° + 4.8\ in \times \cos 60.92° = 9.3\ in$$

For the bearing face:

$$P_u = 248.5\ kips$$

$$v = 0.70$$

$$f_{cu} = mvf'_c = 1.0 \times 0.70 \times 6.0\ ksi = 4.20\ ksi$$

$$\phi P_n = \phi mvf'_c A_{cn}$$

$$\phi P_n = 0.7 \times 4.20\ ksi \times 34.0\ in \times 8.0\ in = 799.7\ kips$$

$$799.7\ kips > 248.5\ kips \quad \boldsymbol{OK}$$

For the strut-to-node interface:

$$m = 1.0$$

$$P_u = 284.4\ kips$$

$$v = 0.85 - \frac{6.0\ ksi}{20.0\ ksi} = 0.55$$

$$f_{cu} = mvf'_c = 1.0 \times 0.55 \times 6.0\ ksi = 3.30\ ksi$$

$$\phi P_n = \phi mvf'_c A_{cn}$$

$$\phi P_n = 0.7 \times 3.30\ ksi \times 34.0\ in \times 9.3\ in = 730.4\ kips$$

$$730.4\ kips > 284.4\ kips \quad \boldsymbol{OK}$$

No direct compressive force acts on the back face of the node; therefore, checking the back face is not necessary.

Hence, Nodes C_s and F_s are sufficient to resist the applied forces.

Other Nodes:

Nodes *D*, *E*, *J, A'* and *F'* of the global strut-and-tie model shown in Figure 3-10 may be checked using the methods presented previously. All of the nodes in the global strut-and-tie model have sufficient strength to resist the applied forces for the studied load case.

Nodes *B* and *H* in the global strut-and-tie model are smeared nodes (interior nodes with no definable geometry); hence, no strength checks are required. Nodes G_s and H_s in the local strut-and-tie models are also smeared nodes and require no strength checks. By inspection, the struts entering these nodal regions have adequate space to spread out and are therefore deemed not to be critical.

Design Step 10 - Proportion Crack Control Reinforcement

The minimum requirements for crack control reinforcement are now checked and compared against the vertical tie reinforcement that was determined in Design Step 7. To maintain consistency in detailing, No. 6 stirrups will be used for the cap beam.

The required spacing of the crack control reinforcement is found by applying the provisions of *AASHTO LRFD* Article 5.8.2.6. The value of b_w is taken as 40.0 in *AASHTO LRFD* Equations 5.8.2.6-1 and 5.8.2.6-2.

$$\frac{A_v}{b_w s_v} \geq 0.003$$

$$s_v \leq \frac{A_v}{0.003 b_w} = \frac{2\ legs \times 0.44\ \frac{in^2}{leg}}{0.003 \times 40.0\ in} = 7.3\ in$$

Recall that the stirrup spacing specified for Tie *CI* at Beam Line 1 will be used for the entire length of the ledge, except in the region surrounding Tie *EK*, where bundled stirrups are used (4-legged stirrups versus 2-legged stirrups). The minimum stirrup spacing required for strength requirements was 5.7 in, therefore the stirrups provided in the ledge region will satisfy the crack control reinforcement requirement.

However, the required crack control reinforcement governs in the region of Tie *BH* and must also be provided over the remaining length of the bent cap beam (i.e., over the columns).

The required spacing of No. 6 reinforcing bars provided as skin reinforcement parallel to the length of the bent cap beam is calculated next using *AASHTO LRFD* Equation 5.8.2.6-2:

$$\frac{A_h}{b_w s_h} \geq 0.003$$

$$s_h \leq \frac{A_h}{0.003 b_w} = \frac{2\ legs \times 0.44\ \frac{in^2}{leg}}{0.003 \times 40.0\ in} = 7.3\ in$$

Skin reinforcement of 2 No. 6 reinforcing bars will be provided at a spacing of less than 7.3 in.

Design Step 11 - Provide Necessary Anchorage for Ties

The reinforcement in the top and bottom chords of the global strut-and-tie model must be properly anchored at each end of the bent cap beam in accordance with *AASHTO LRFD* Article 5.10.8.2. Continuity of the reinforcement over the length of the bent cap beam will be provided by using longitudinal lap splices. Proper anchorage of the horizontal ledge reinforcement in the local strut-and-tie models must also be ensured.

The bottom chord reinforcement of the cap beam must be fully developed at Nodes *G* and *L*. If straight reinforcing bars are to be used, the required tension development length is determined using *AASHTO LRFD* Equation 5.10.8.2.1a-1:

$$l_d = l_{db} \times \left(\frac{\lambda_{rl} \times \lambda_{cf} \times \lambda_{rc} \times \lambda_{er}}{\lambda}\right)$$

where l_{db} is the *basic development length*, defined by *AASHTO LRFD* Equation 5.10.8.2.1a-2:

$$l_{db} = 2.4 d_b \frac{F_y}{\sqrt{f'_c}}$$

The required development length will be compared against the available development length at the ends of the bent cap beam, as shown in Figure 3-27 below. In Figure 3-27, it is determined that the available development length is approximately 69.4 in.

Examining *AASHTO LRFD* Article 5.10.8.2.1b, no modification factors are required which would increase the required development length (uncoated, or black, reinforcing bars are assumed). Conservatively, the calculated development length will not be reduced as allowed by *AASHTO LRFD* Article 5.10.8.2.1c. Thus:

$$l_d = 2.4 d_b \frac{F_y}{\sqrt{f'_c}} \times \left(\frac{\lambda_{rl} \times \lambda_{cf} \times \lambda_{rc} \times \lambda_{er}}{\lambda}\right)$$

$$l_d = 2.4 \times 1.410\ in \times \frac{60.0\ ksi}{\sqrt{6.0\ ksi}} \times \left(\frac{1.0 \times 1.0 \times 1.0 \times 1.0}{1.0}\right) = 82.9\ in$$

$$82.9\ in > 69.4\ in \quad \textbf{\textit{NO GOOD}}$$

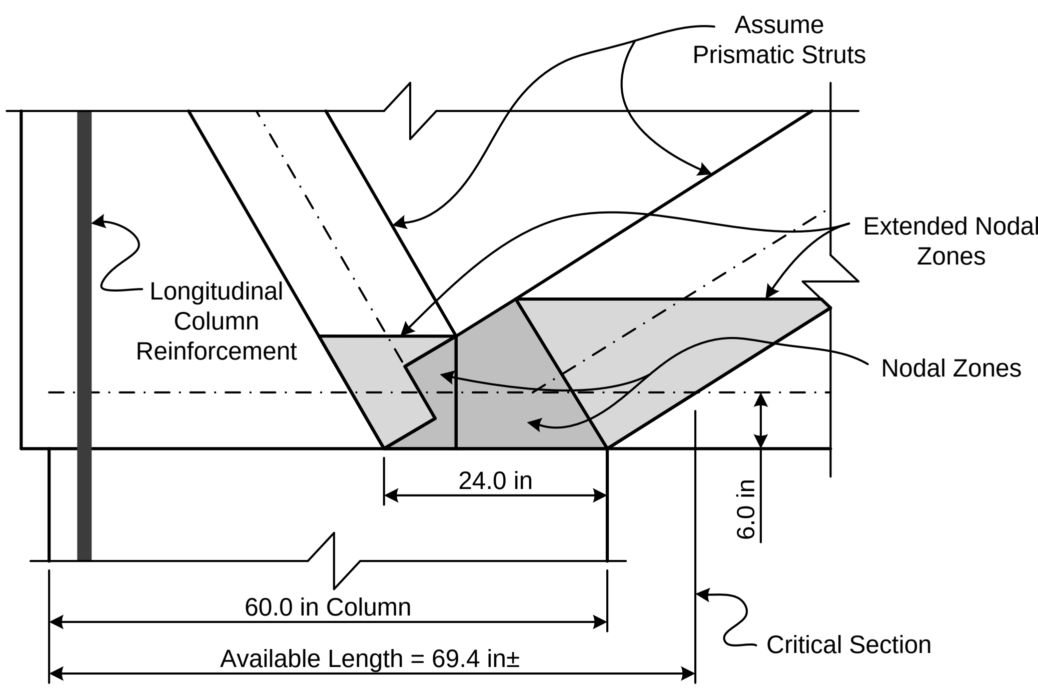


Figure 3-27: Bottom Chord Reinforcement Anchorage at Node *G*

The required development length is greater than the available length as illustrated in Figure 3-27. Therefore, try providing hooked ends and checking the required development length for a hooked bar. The required development length of a hooked bar is given by *AASHTO LRFD* Equation 5.10.8.2.4a-1:

$$l_{dh} = l_{hb} \times \left(\frac{\lambda_{rc}\lambda_{cw}\lambda_{er}}{\lambda}\right)$$

where l_{hb} is given by *AASHTO LRFD* Equation 5.10.8.2.4a-2:

$$l_{hb} = \frac{38d_b}{60} \times \left(\frac{f_y}{\sqrt{f_c'}}\right)$$

Consequently:

$$l_{dh} = \frac{38 \times 1.410\ in}{60} \times \left(\frac{60.0\ ksi}{\sqrt{6.0\ ksi}}\right) \times \left(\frac{1.0 \times 1.0 \times 1.0}{1.0}\right) = 21.9\ in$$

$$21.9\ in < 69.4\ in \quad \boldsymbol{OK}$$

Adequate length is available to develop a hooked reinforcing bar in tension; therefore, provide standard hooks at the ends of the bottom chord reinforcing.

Proper development of the top chord reinforcing is satisfied by the fact that the top chord reinforcing is continuous around the corners of the moment frame, and by inspection it will be adequately developed at Nodes *A* and *F*.

Finally, proper anchorage of the ledge reinforcement (Tie C_sF_s of the local strut-and-tie models) must be checked. The top horizontal portion of the ledge reinforcement is terminated in a 90-degree hook. Recall that the available development length at Nodes C_s and F_s of the local strut-and-tie model is measured from the location where the centroid of the reinforcing enters the extended nodal zone, illustrated in Figure 3-28 on the following page.

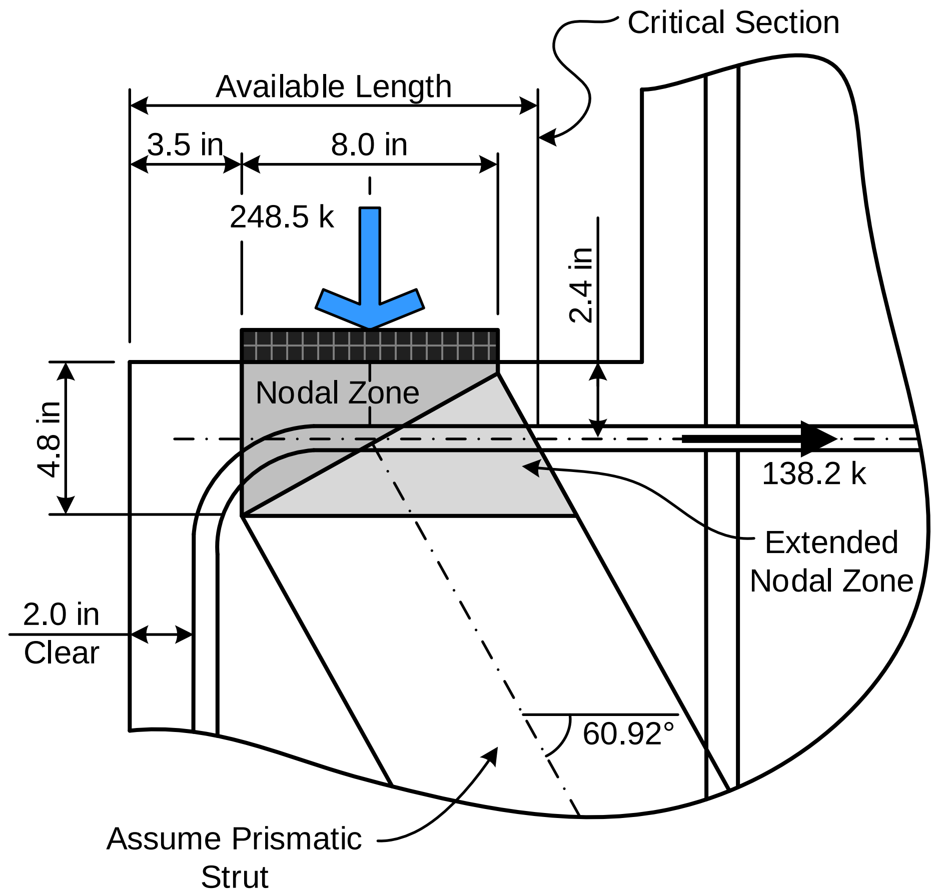


Figure 3-28: Ledge Reinforcement Anchorage at Node *C_s*

The available development length is:

$$l_{d,available} = 3.5\ in + 8.0\ in + \frac{2.4\ in}{\tan 60.92°} - 2.0\ in = 10.8\ in$$

The required development length of a No. 6 reinforcing bar with a 90-degree hook is:

$$l_{dh} = \frac{38 d_b}{60} \times \left(\frac{f_y}{\sqrt{f_c'}}\right) \times \left(\frac{\lambda_{rc}\lambda_{cw}\lambda_{er}}{\lambda}\right)$$

$$l_{dh} = \frac{38 \times 0.75\ in}{60} \times \left(\frac{60.0\ ksi}{\sqrt{6.0\ ksi}}\right) \times \left(\frac{0.8 \times 1.0 \times 1.0}{1.0}\right) = 9.3\ in$$

$$9.3\ in < 10.8\ in \quad \boldsymbol{OK}$$

Note that the reinforcement confinement factor, λ_{rc}, is taken as 0.8 per *AASHTO LRFD* Article 5.10.8.2.4b. Sufficient development length is available for the ledge reinforcing using 90-degree hooked ends.

Design Step 12 - Draw Reinforcement Layout

At this point, the strut-and-tie analysis for the example moment frame inverted-tee straddle bent cap beam is complete. The designer is reminded that other load cases must be checked to ensure adequate strength is provided for all imposed loads. Sketches of the final beam dimensions and reinforcing layouts follow. The designer must also consider reinforcing details such as reinforcing lap splice locations, possible reinforcing conflicts (such as between the column reinforcement and bottom mat of reinforcement in the cap beam), and minimum reinforcement spacing.

Specific Agency Policies and Practices:

Any reinforcement details not shown in the following figures that were not described within this design example may be adjusted based on specific state or agency policies and practices.

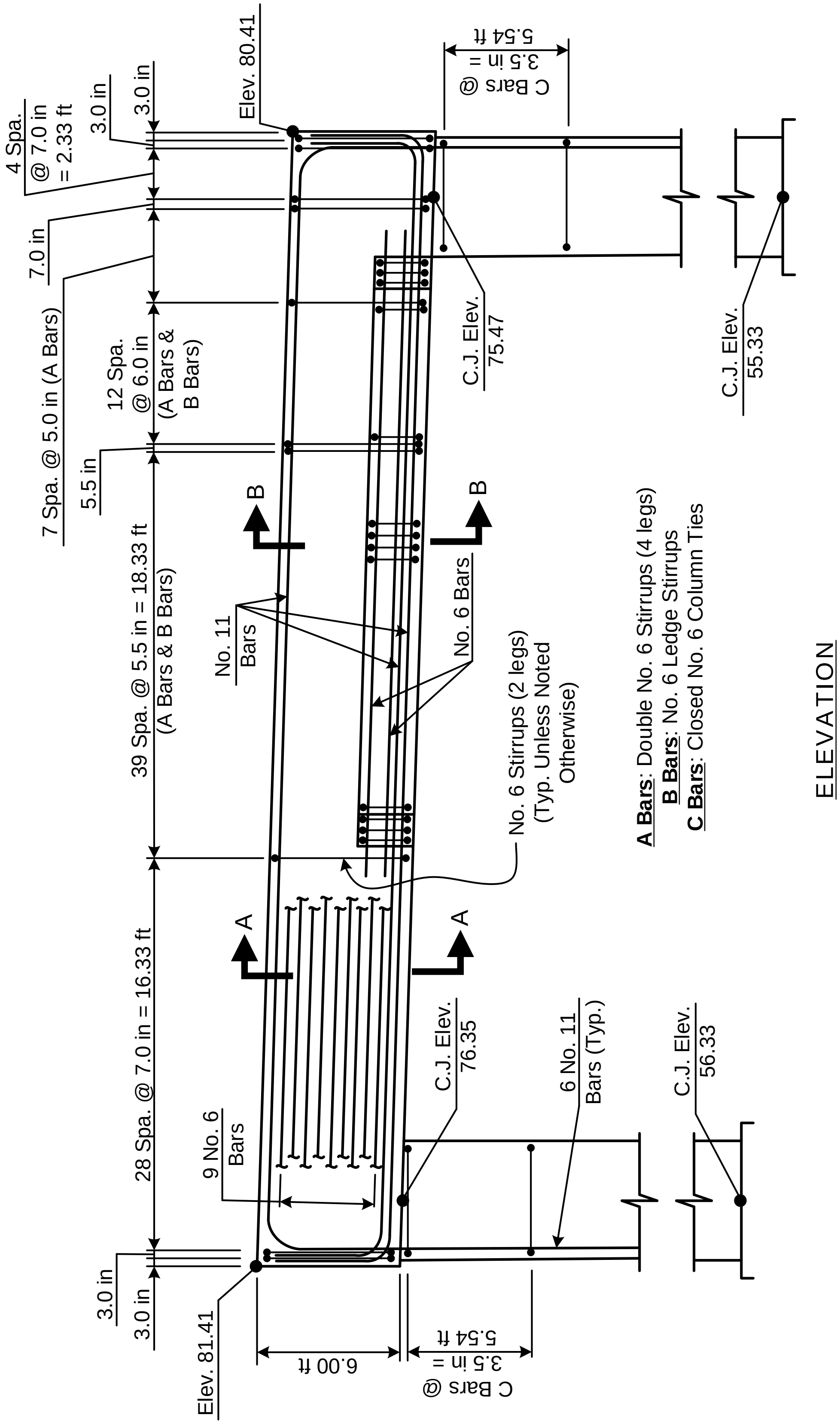


Figure 3-29: Elevation View of Reinforcement Layout

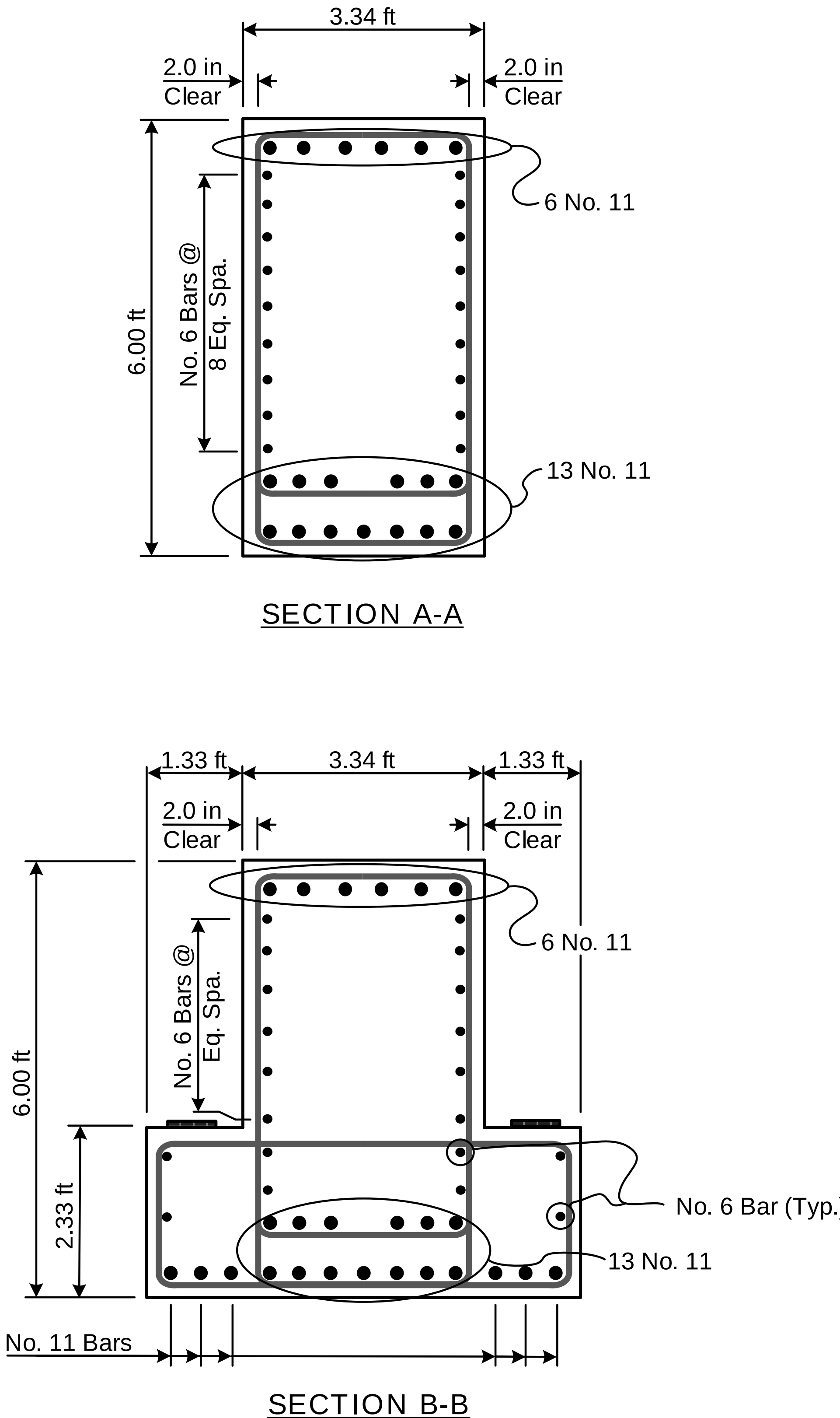


Figure 3-30: Sections A-A and B-B of Figure 3-29

References:

1. Williams, C.S., Deschenes, D.J., and Bayrak, O., *Strut-and-Tie Model Design Examples for Bridges*, Implementation Report 5-5253-01, Center for Transportation Research, Bureau of Engineering Research, University of Texas at Austin, October 2011, 272 pp.

Design Example #4 – Drilled Shaft Footing

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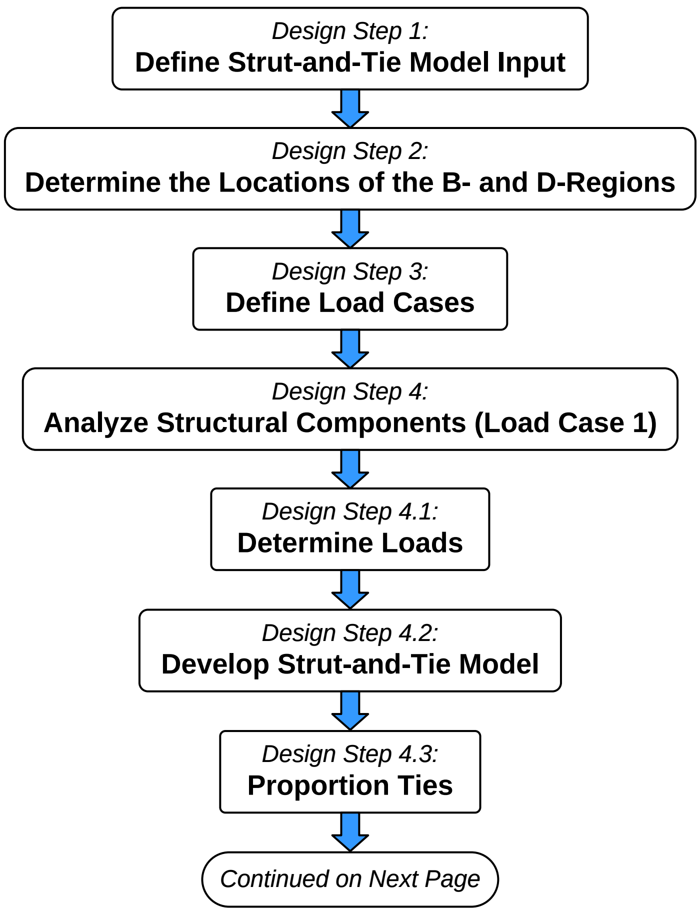
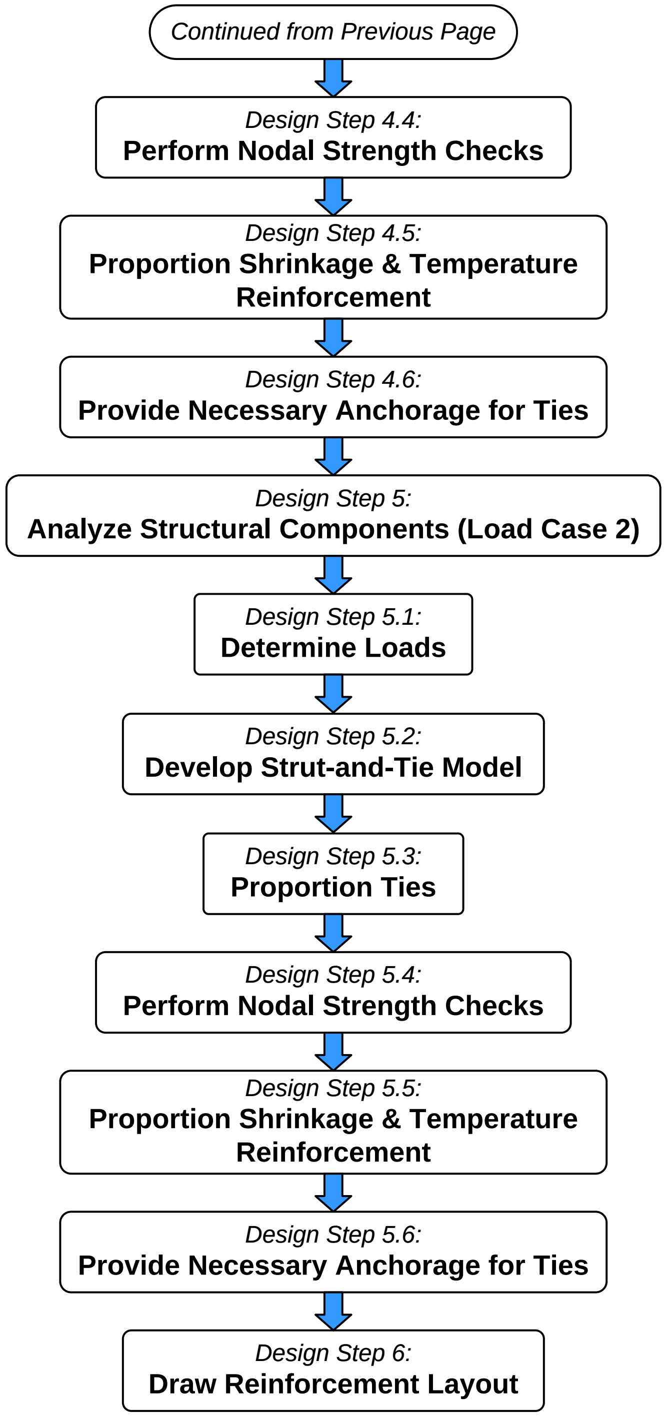
Design Example #4 presents the application of the strut-and-tie method (STM) to the analysis and design of a drilled shaft footing. The footing supports a single column and is supported by four drilled shafts. Existing research demonstrates that the strut-and-tie method is appropriate for the design of footings such as these (see the references at the end of this design example). This design example demonstrates the development and application of three-dimensional strut-and-tie models to effectively model the complex distribution of forces in deep footings. The example features the elements of strut-and-tie design of concrete members listed below:





Please note that this example is based on an example problem developed in an implementation project sponsored by TxDOT (Report No. 5-5253-01, Williams et al., 2012). Figures included in this design example are adapted from this report. The example has been revised herein to provide additional explanations and to provide compliance with the STM provisions of the 8th Edition of *AASHTO LRFD*, as appropriate, and differing material strengths have been adopted for reasons that will be discussed shortly.

There is a shortage of documented research on the application of the strut-and-tie design method to the design of deep pile caps or drilled shaft footings. As a result, several approximations and assumptions must be made to design these structures. The implications of these assumptions on the design are analyzed and discussed before they are used; these judgements tend to err on the side of conservatism.

Design Step 1 - Define Strut-and-Tie Model Input

Figure 4-1 illustrates the overall geometry of the drilled shaft footing under consideration. The footing is determined to be 5.00 ft thick, 16.00 ft wide, and 16.00 ft long. The function of the footing is to transfer loads imposed by a 7.50 ft by 6.25 ft rectangular column to four drilled shafts that are each 4.00 ft in diameter. This means of load transfer provides an opportunity to demonstrate the *AASHTO LRFD* strut-and-tie design specifications in a three-dimensional context.

The design compressive strength of the concrete, f'_c, is taken as 5.0 ksi, and the yield strength of the steel reinforcing, f_y, is taken as 75.0 ksi. The compressive strength of the concrete represents an average of concrete compressive strengths for footings used nationally and differs from the original design example developed by Williams et al. (2012). The reinforcement strength is chosen to be 75.0 ksi in order to take advantage of smaller required reinforcing areas. The *AASHTO LRFD* design specifications permit the use of higher-strength reinforcing bars (strengths in excess of 60.0 ksi). Grade 75 reinforcing is used in this example to illustrate the use of the *AASHTO LRFD* design specifications with high-strength material that is now available. The appropriate strengths should be chosen per the Owner's/Agency's requirements.

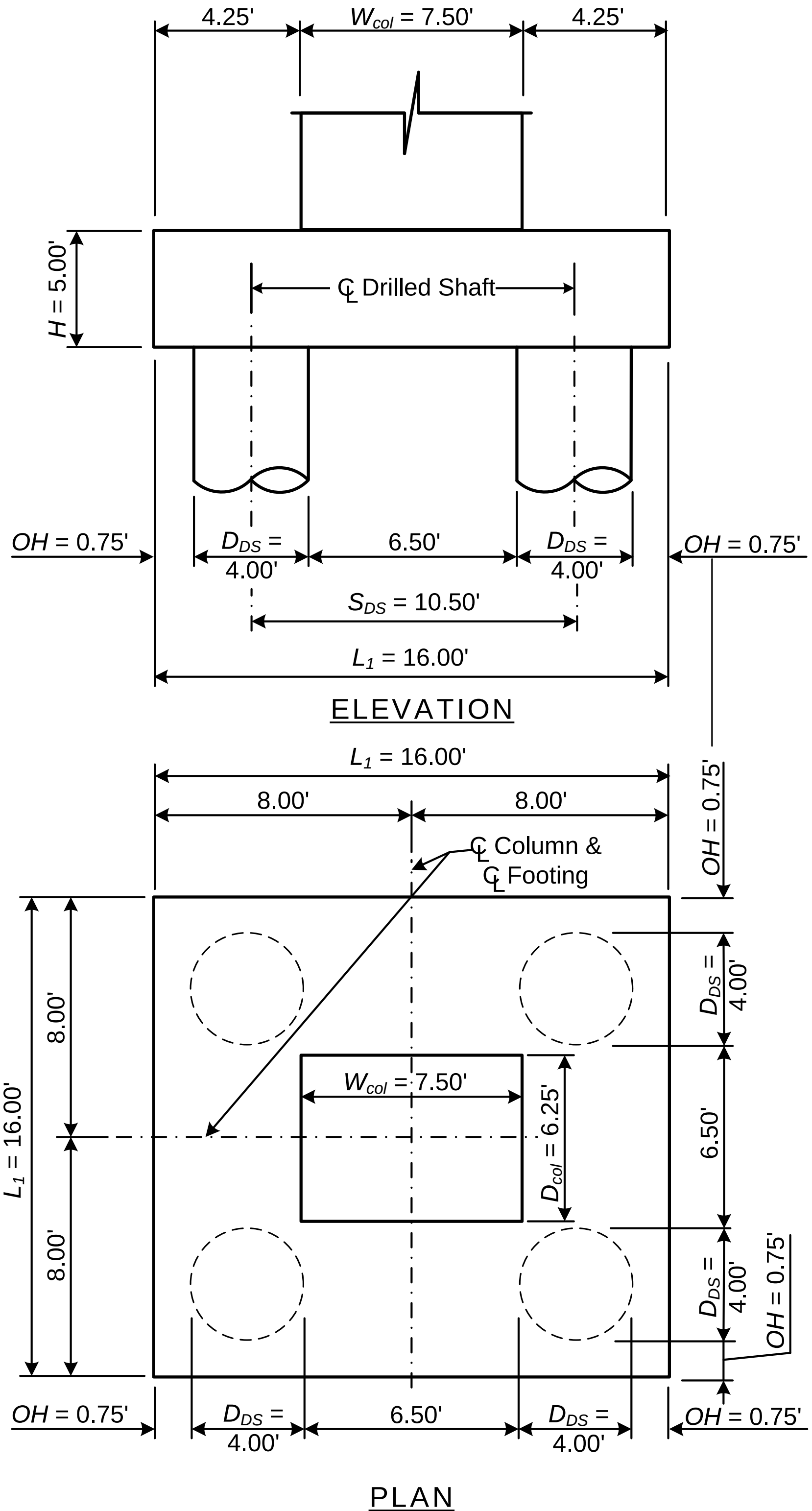


Figure 4-1: Plan and Elevation Views of Drilled Shaft Footing (Adapted from Williams et al., 2012)

Design Step 2 - Determine the Locations of the B- and D-Regions

The subject drilled shaft footing is characterized by disturbances introduced by the column and drilled shafts. Because of their close proximity (relative to the depth of the footing itself), the classical Bernoulli beam theory assumption that “plane sections remain plane” is inappropriate. As such, the entire footing is classified as a D-Region.

Design Step 3 - Define Load Cases

In a departure from previous design examples, this design example presents two load cases. Each consists of a system of forces imposed by the column on the drilled shaft footing, which are in turn resisted by the drilled shafts. In Load Case 1, the column is subjected to combination of axial force and strong-axis bending moment (refer to Figure 4-2). When the load is transferred through the footing, all of the drilled shafts will remain in compression, as shown in Figure 4-2. The actual calculation of the support reactions is discussed in Design Step 4.1.

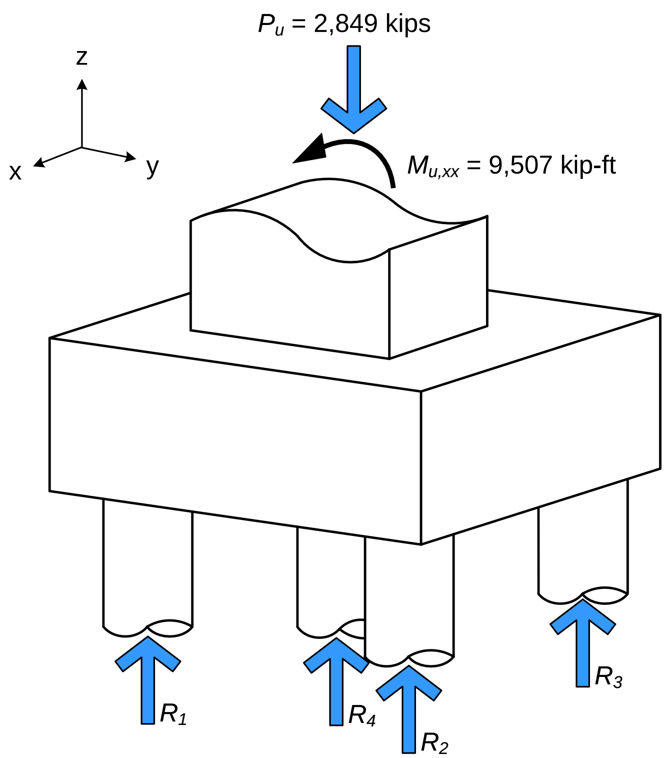


Figure 4-2: Factored Loads for Load Case 1 (Adapted from Williams et al., 2012)

In Load Case 2, the column is again subjected to axial force and strong-axis bending moment (see Figure 4-3). However, in this case, the strong-axis bending moment is approximately 16 percent less than that in Load Case 1, and the magnitude of the axial force is less than half that in Load Case 1. This load case will result in tension in two of the drilled shafts, as shown in Figure 4-3. The calculation of these reactions is discussed in Design Step 5.1.

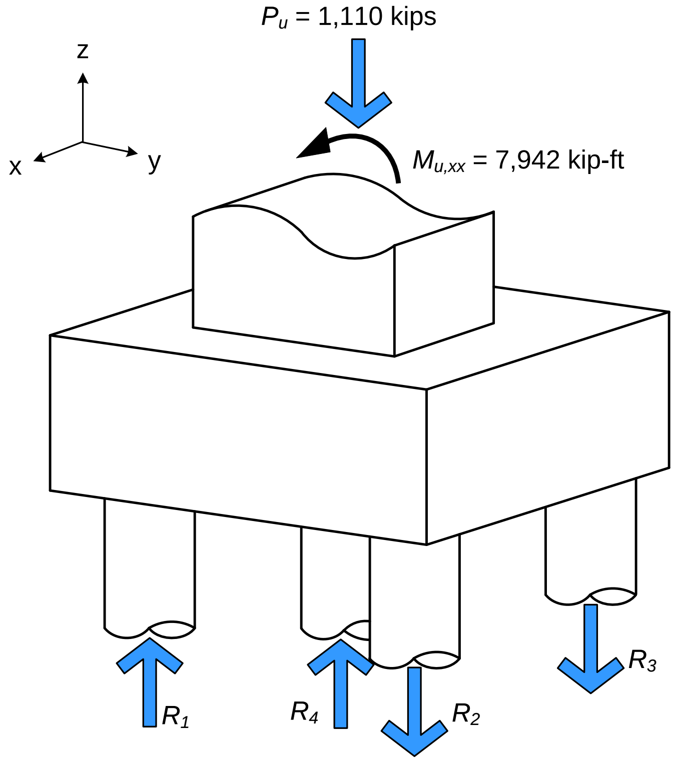


Figure 4-3: Factored Loads for Load Case 2 (Adapted from Williams et al., 2012)

Many designers include the footing self-weight due to the possibility of earth settlement. However, for this design example, the possibility of earth settlement is considered to be insignificant, and the footing self-weight is not applied in either load case. Each load case is presented independently; i.e. all of the design steps will be performed for Load Case 1 before performing design for Load Case 2.

> *Evaluating Multiple Load Cases:*
>
> This design example considers only the two load cases presented. The designer is reminded that a complete design of the drilled shaft footing is contingent on examination of all of the critical load cases.

Design Step 4 - Analyze Structural Components (Load Case 1)

The forces imposed by the column will flow through the footing to each of the four drilled shafts. In order to properly model the flow of forces, the axial force and moment applied by the column on the footing must be rectified into a system of equivalent forces (as was done for the cantilever bent in Design Example 2 and the straddle bent columns in Design Example 3). This set of forces will be applied to the strut-and-tie model and should, by definition, produce the same axial load and moment as shown in Figure 4-2. An illustration of this procedure is given in Figure 4-4.

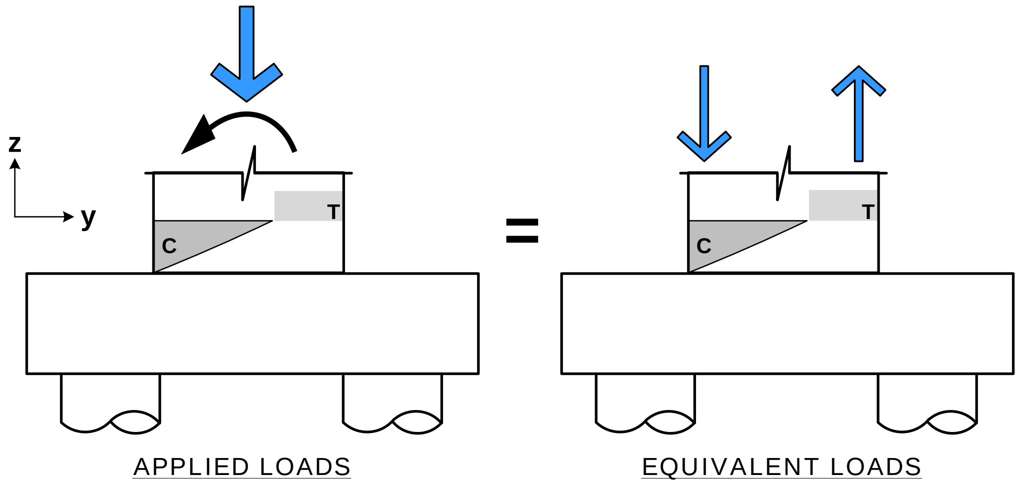


Figure 4-4: Developing an Equivalent Force System

Design Step 4.1 - Determine Loads

To develop the system of equivalent forces, the elastic stress distribution within the column must be determined. The location of each of the forces in the equivalent force system is found relative to the column cross-section. The assumed layout of the column reinforcing is given in Figure 4-5. Then, the magnitude of each force may be found by establishing equilibrium. The column cross-section and corresponding linear stress distribution are shown in Figure 4-6. The positions of the four loads that will comprise the equivalent force system are shown. The two loads acting on the left side of the column are compressive (pushing down on the footing) and the two loads acting on the right of the column are tensile (pulling up on the footing).

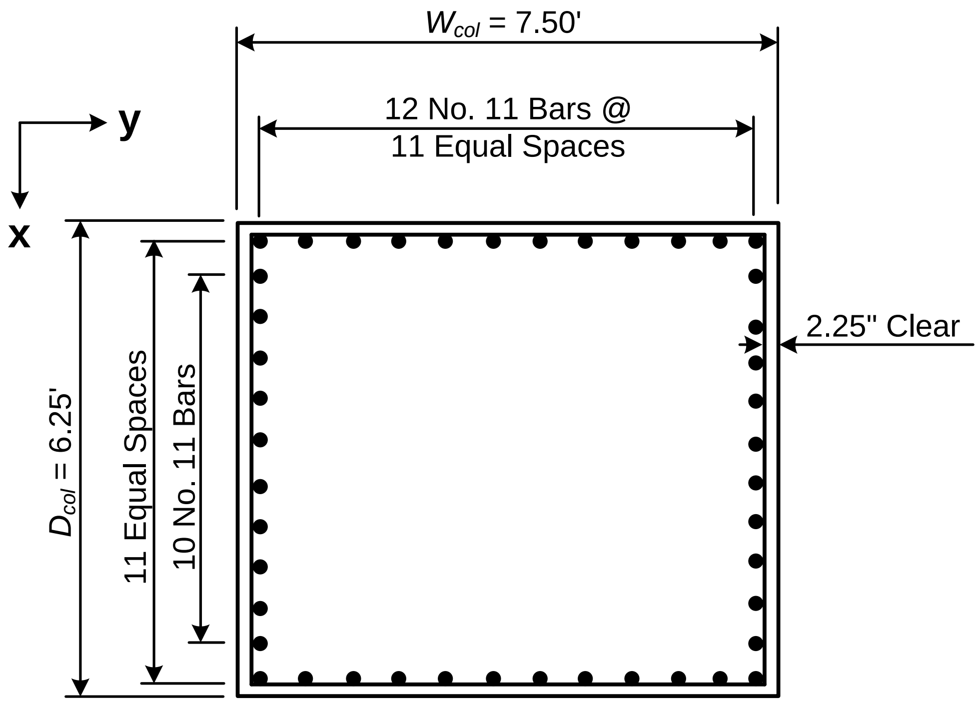


Figure 4-5: Preliminary Column Reinforcing Layout

The locations of the compressive forces are based on the linear stress diagram. The line of action for the compressive forces coincides with the location of the center of gravity of the compressive side of the stress diagram. This line of action is located 1.72 ft from the left face of the column. The loads are located transversely at the quarter-points of the column depth (6.25 ft), which is 1.56 ft from the top and bottom of the column section in Figure 4-6.

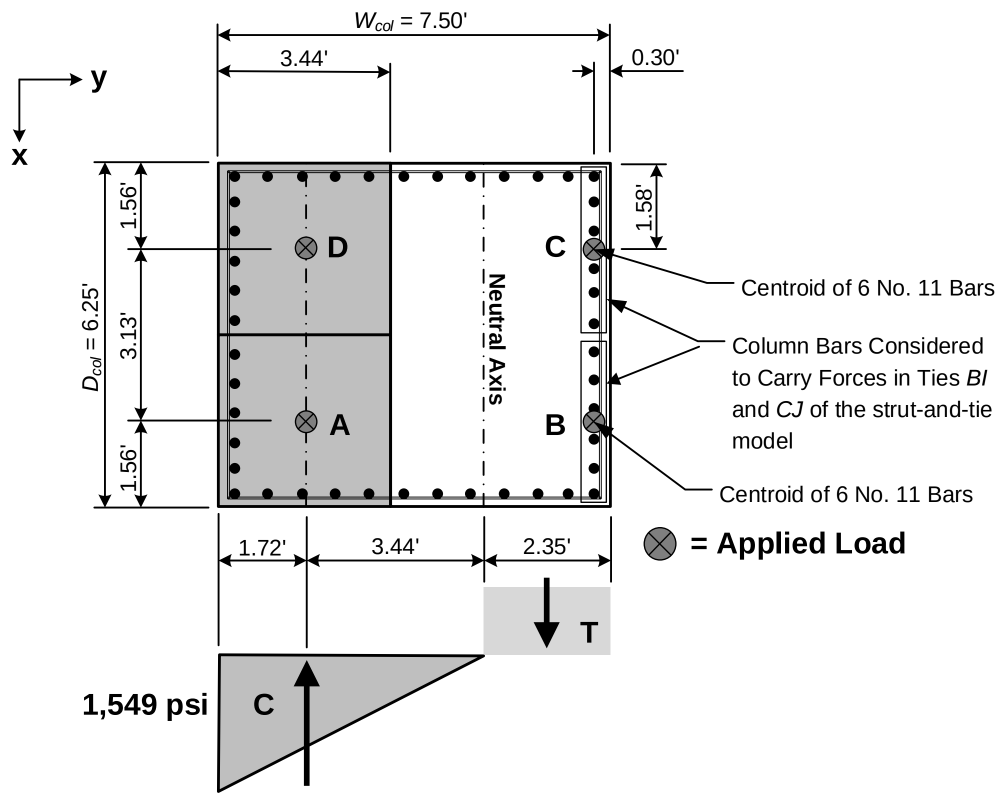


Figure 4-6: Linear Stress Distribution over Column Cross-Section and Equivalent Force System Load Locations

The longitudinal reinforcing steel in the column shown in Figure 4-5 and Figure 4-6 is an assumption. The size and configuration of the reinforcement must be determined in final design, which is beyond the scope of this design example. The reinforcement on the right face of the column will resist the tension resulting from the applied bending moment. The two tensile forces which will balance the two compressive forces are therefore located at the centers of gravity of the tension-face reinforcement, located at 0.30 ft from the right face of the column. The reinforcing on this face is divided in half, and each half is assumed to resist each tensile force. This results in each tensile force being located at the center of gravity of one-half of the tension-face reinforcement. As shown in Figure 4-6, each tie in the column will consist of six reinforcing bars.

The magnitudes of the compressive and tensile forces must now be determined so that the equivalent force system produces the same axial force, moment, and linear stress distribution. This is accomplished by establishing equilibrium between the original and equivalent force systems. In the following system of equations, F_{comp} is the total compressive force acting on the system (or, the sum of the compressive forces acting at Points *A* and *D*) and F_{tens} is the total tensile force acting on the system (or, the sum of the tensile forces acting at Points *B* and *C*).

$$\begin{cases} F_{comp} & -F_{tens} & = 2{,}849\ kips \\ \left(\dfrac{7.50\ ft}{2} - 1.72\ ft\right) F_{comp} & + \left(\dfrac{7.50\ ft}{2} - 0.30\ ft\right) F_{tens} & = 9{,}507\ kip - ft \end{cases}$$

Solving yields:

$$F_{comp} = 3{,}527.1\ kips$$

$$F_{tens} = 678.1\ kips$$

where 7.50 ft is the width of the column, and 1.72 ft and 0.30 ft are the distances to the centers of gravity of the compressive and tensile forces, respectively. Therefore, the four loads that will act on the strut-and-tie model from the column are determined:

$$F_A = F_D = \frac{F_{comp}}{2} = 1{,}763.6\ kips$$

$$F_B = F_C = \frac{F_{tens}}{2} = 339.1\ kips$$

The footing must now be analyzed to determine the reaction forces (the forces in each drilled shaft). The reactions are assumed to act at the center of each of the drilled shafts. Since all four drilled shafts are spaced equidistant from the column, the axial force is assumed to be distributed equally to each drilled shaft. Moment equilibrium of the footing is enforced by equating the column moment to each drilled shaft's axial force times its orthogonal distance from the column (see Figure 4-6). The drilled shaft reactions are determined thus:

$$R_1 = R_4 = \frac{P_u}{4} + \frac{1}{2}\left(\frac{M_{u,xx}}{s_{Ds}}\right) = \frac{2{,}849\ kips}{4} + \frac{1}{2} \times \frac{9{,}507\ kip - ft}{10.50\ ft} = 1{,}165.0\ kips$$

$$R_2 = R_3 = \frac{P_u}{4} - \frac{1}{2}\left(\frac{M_{u,xx}}{s_{Ds}}\right) = \frac{2{,}849\ kips}{4} - \frac{1}{2} \times \frac{9{,}507\ kip - ft}{10.50\ ft} = 259.5\ kips$$

where the value of 10.50 ft is the drilled shaft spacing parallel to the applied moment. Note that all drilled shaft reactions are of the same sign, indicating that they act in the same direction. By inspection, all of the drilled shaft reactions are compressive.

Design Step 4.2 - Develop Strut-and-Tie Model

The strut-and-tie model for Load Case 1 is shown in Figure 4-8 and Figure 4-9. Figure 4-8 should be worked with Figure 4-9, and the coordinates of each node in the strut-and-tie model are presented in Table 4-1. Development of the three-dimensional strut-and-tie model is considered successful only if equilibrium is satisfied at every node, and the truss reactions (determined from a linear elastic analysis of the truss model) are equivalent to the reactions found in Design Step 4.1. In order to successfully develop the three-dimensional truss model, the designer first must determine the lateral (*x*, *y*) location of each applied load and support reaction. Then, the designer must determine the vertical (*z*) position of the planes where the upper and lower nodes of the strut-and-tie model lie. The lateral locations of the applied loads and drilled shaft reactions were determined in Design Step 4.1.

Table 4-1: Coordinates of Nodes in Strut-and-Tie Model for Load Case 1

Node	x-Coordinate	y-Coordinate	z-Coordinate
A	9.57’	5.97’	4.50’
B	9.57’	11.45’	4.50’
C	6.44’	11.45’	4.50’
D	6.44’	5.97’	4.50’
E	13.25’	2.75’	0.45’
F	13.25’	13.25’	0.45’
G	2.75’	13.25’	0.45’
H	2.75’	2.75’	0.45’
I	9.57’	11.45’	0.45’
J	6.44’	11.45’	0.45’

Note: The origin is located in the bottom corner of the footing nearest to Node *H*.

The location of the bottom horizontal ties relative to the bottom of the footing are determined first. These ties (Ties EF, FG, GH, and EH) represent the bottom mat of reinforcing steel within the footing. Their locations should therefore be based on the location of the center of gravity of the reinforcing. Four inches of clear cover will be provided from the bottom of the footing to the bottom layer of reinforcing, as shown in Figure 4-**7**. Assuming the same number of reinforcing bars will be used in both directions and No. 11 reinforcing bars will be used, the center of gravity of the bottom mat of reinforcing is located at:

$$z_{bottom\ mat} = 4.00\ in + 1.41\ in = 5.41\ in$$

above the bottom face of the footing.

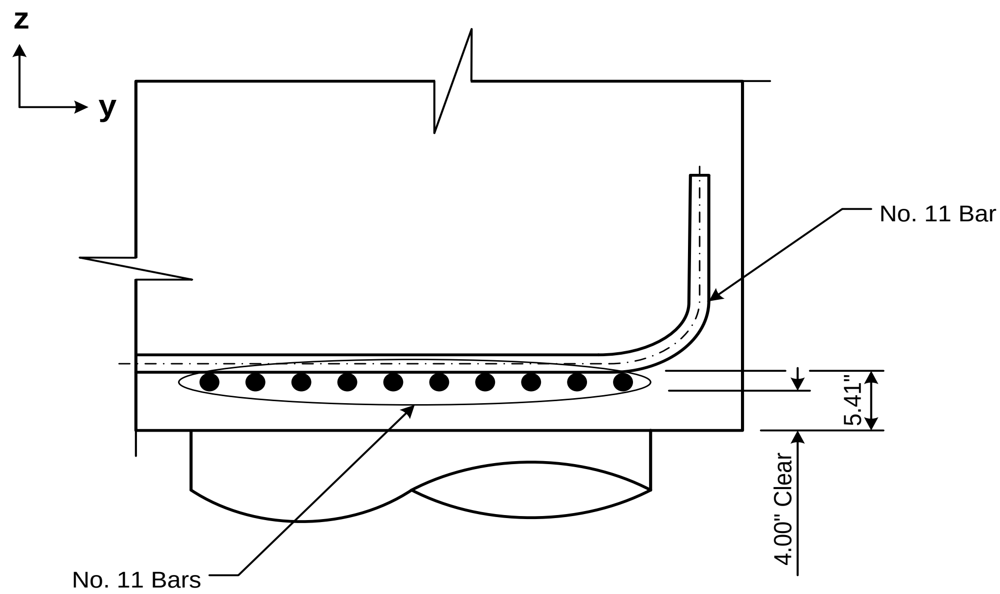


Figure 4-7: Location of Bottom Mat of Reinforcing

Nodes *I* and *J* are located in the same plane as Nodes *E*, *F*, *G*, and *H*. Based on the analysis for this load case, Members *IF* and *JG* are struts since they must balance the horizontal compression at Nodes *I* and *J* due to Struts *AI* and *DJ*, respectively. The location of Nodes *E*, *F*, *G*, *H*, *I*, and *J* coincides with the plane of the bottom mat of reinforcing steel in the shaft cap.

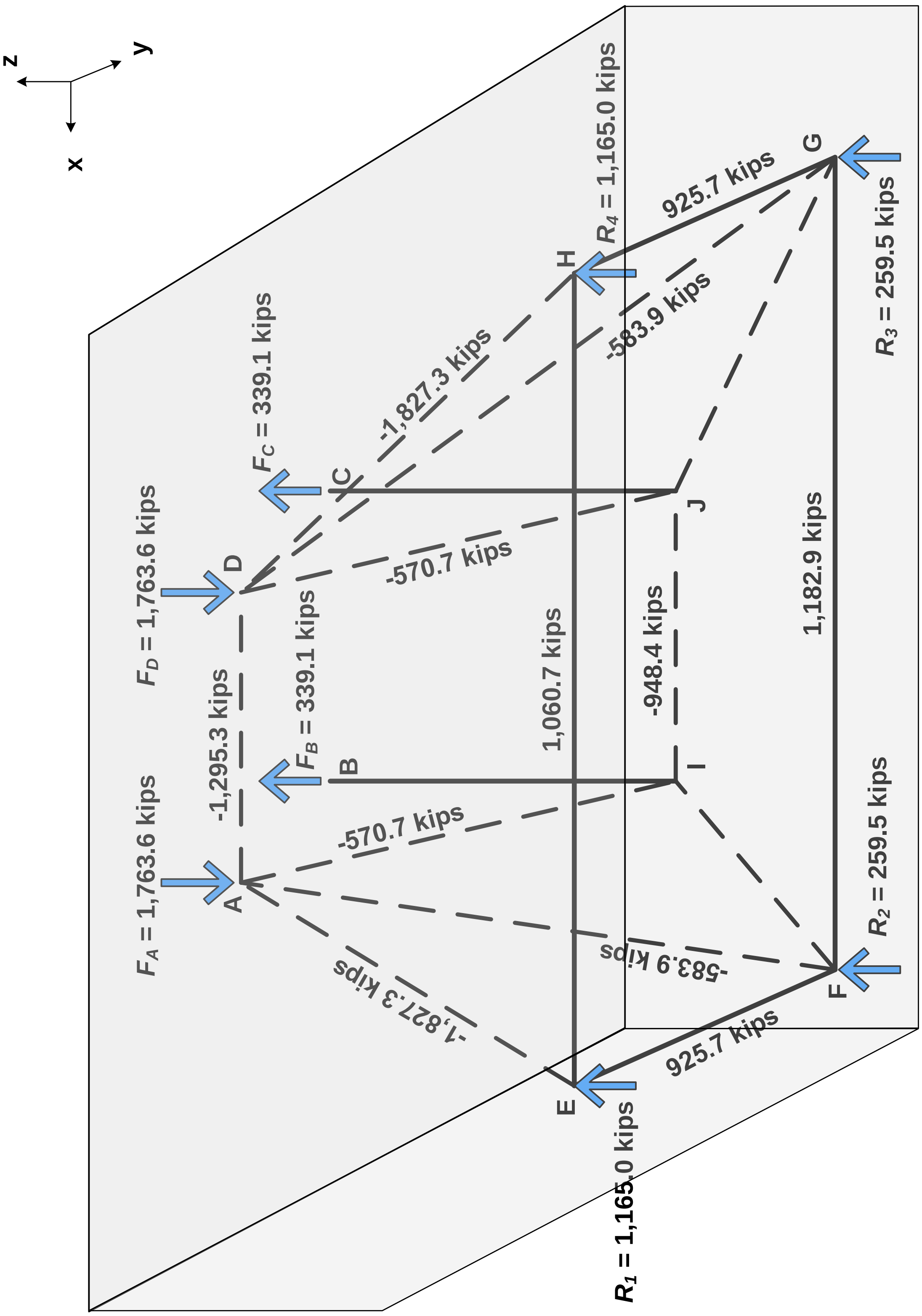


Figure 4-8: Isometric View of Strut-and-Tie Model for Load Case 1 (Work This Figure with Figure 4-9)

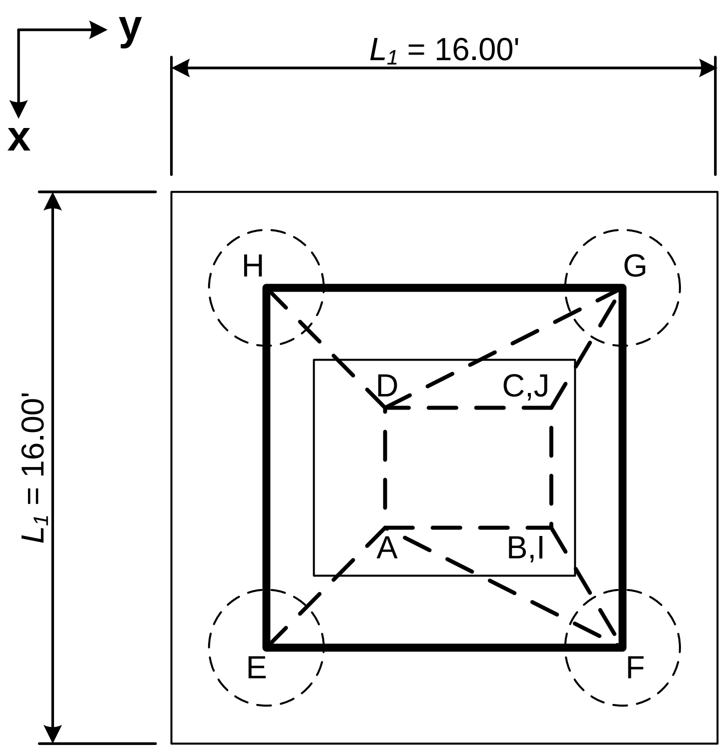


Figure 4-9: Plan View of Strut-and-Tie Model for Load Case 1 (Work This Figure with Figure 4-8)

The distance between the horizontal strut (Strut *AD*) and the top face of the footing may be determined in several ways. Williams et al. (2012) reports a detailed discussion on this issue, based on their research. Existing research on this topic recommends different locations of the top struts. The options discussed in Williams et al. (2012) include:

- ***Option 1: Position Nodes* A *and* D *at the top surface of the footing***
  - This is an obvious choice for the node locations; however, effective triaxial confinement of the nodes cannot be guaranteed and more conservative estimates of the nodal strength must be used
  - Additionally, positioning the nodes at the top of the footing will create an artificially deep strut-and-tie model (increasing h_{STM}) which would result in under-predicting the bottom tie forces

- ***Option 2: Assume depth of the horizontal Strut* AD *is equal to (*h*/4) where* h *is the overall depth of the footing***
  - This would place the center of the strut at *h*/8 from the top face of the footing

  - This option is recommended in Park et al. (2008) and in Windisch et al. (2010)
  - This option is particularly applicable for the depth of a flexural compression zone of an elastic column at a beam-column joint
  - However, this option is not highly applicable for this specific design example

- ***Option 3: Position the Nodes* A *and* D *based on the depth of the Whitney compression stress block***
  - This would determine the strut locations based on the depth of the compression zone determined by a flexural (beam theory) analysis of the footing
  - Because the footing is a deep member with many loads and disturbances, it is subject to very nonlinear strain distributions, so applying a beam theory analogy would not be appropriate
  - This approach was used in other design examples; however, it is not used for this design example, because the strains are not only nonlinear but they are nonlinear in three dimensions

- ***Option 4: Align Nodes* A *and* D *with the location of the top mat of reinforcing***
  - If horizontal ties were located in the top of the footing, this would be another viable location for the nodes
  - This method is used to develop the strut-and-tie model for Load Case 2

To summarize, there are numerous options that the designer may consider when placing the top struts. The designer may consider the above options, but can also consider other options that apply to the specific design and that will result in a conservative design. Consideration should also be given to the fact that moving the nodes deeper into the footing (farther from the top surface) decreases the effective height of the strut-and-tie model, which will increase the demands in the bottom ties. This will also increase the effect of the triaxial confinement of the nodes.

For the purposes of this design example, 0.1*h* was chosen as the distance between the top chord (compression) of the space truss and the top face, as shown in Figure 4-10. This value was selected for several reasons:

- A value of 0.1*h* is anticipated to produce conservative results
- A value of 0.1*h* is within the lower and upper bound of the four options presented above
- A value of 0.1*h* results in a “clean” value of 6 inches

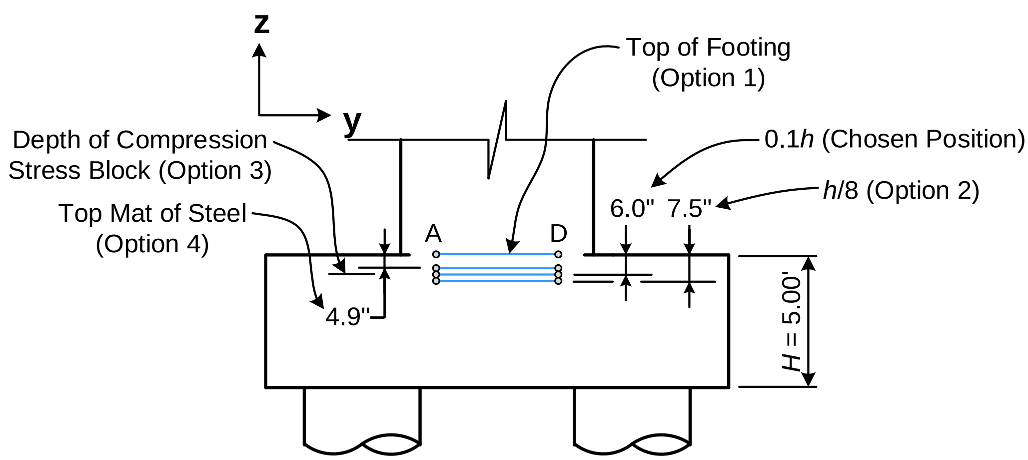


Figure 4-10: Locations of Ties in Top of Footing

In summary, the distance from the bottom horizontal ties of the strut-and-tie model to the bottom of the footing is taken as 5.41 in, and the distance from the top of the footing to Nodes *A* and *D* is assumed to be 6.00 in. Thus, the total height of the strut-and-tie model is:

$$h_{STM} = 60.00\ in - 5.41\ in - 6.00\ in = 48.59\ in$$

Defining and Refining the Model:

Defining the basic geometry of the strut-and-tie model may be accomplished reasonably simply. However, establishing the struts and ties within the model can be more difficult. Further refinement of the strut-and-tie model is based upon the following:

- Recognizing the most probable load paths (flow of forces)
- Considering standard concrete construction details
- Understanding the behavior of footings
- Iterating by trial-and-error to establish equilibrium

The logic used to develop the strut-and-tie model for this design example is discussed in Williams et al. (2012) and is presented here for the reader's benefit.

To begin, the tensile forces acting at Nodes *B* and *C* will require vertical ties that pass through the depth of the footing to Nodes *I* and *J*. Although the forces in these ties are simply the column loads, they are included in this model since establishing the requirement of the ties leads into how the geometry of the entire model is developed. The determination of the column ties is presented for completeness.

Ties should almost always be oriented parallel or perpendicular to the primary axes of the structural component, since inclined reinforcement is rarely used in reinforced concrete construction. The forces in these vertical ties must be equilibrated by compressive stresses originating at Nodes *A* and *D*, which leads to the placement of Struts *AI* and *DJ*. In turn, Struts *AE, AF, DG,* and *DH* represent the flow of compressive forces from Nodes *A* and *D* to the supports at Nodes *E, F, G,* and *H*. Finally, equilibrium is established at Node *D* by adding Strut *AD*.

The flow of compressive forces to each of the drilled shafts (at Nodes *E, F, G,* and *H*) will induce tension in the bottom of the footing which must be equilibrated with ties. Thus, Ties *EF, FG, GH,* and *EH* are established. The remaining horizontal struts are added to the bottom of the strut-and-tie model to establish lateral equilibrium at Nodes *F, G, I,* and *J*. As with all strut-and-tie models, recall that the angle between a tie and an adjacent strut must be greater than or equal to 25 degrees to comply with *AASHTO LRFD* Article C5.8.2.2. The strut-and-tie model shown in Figure 4-8 satisfies this requirement.

Three-Dimensional Equilibrium:

The designer is reminded that equilibrium must be maintained in all three orthogonal directions (laterally and vertically) at every node in the strut-and-tie truss model. There should be enough truss members intersecting at each node to maintain equilibrium in the x, y, and z directions, and the designer needs to perform a stability check using a three-dimensional truss model.

As a check, keep in mind that a symmetrical footing geometry and symmetrical loading should result in a symmetrical strut-and-tie model.

Once the strut-and-tie model geometry has been defined, the truss member forces and drilled shaft reactions are determined through a linear elastic analysis, either manually or using a structural analysis software package. The reactions at each of the drilled shaft locations should be equal to the reactions determined in Design Step 4.1 and equilibrium should be satisfied at each node. If equilibrium cannot be established, the strut-and-tie model must be revised and re-analyzed.

Another valid strut-and-tie model is presented in Figure 4-11 below. Although it was possible to establish equilibrium at each node, the overall model does not accurately represent the flow of compressive forces from Nodes *A* and *D* to each of the drilled shafts (at Nodes *E, F, G*, and *H*).

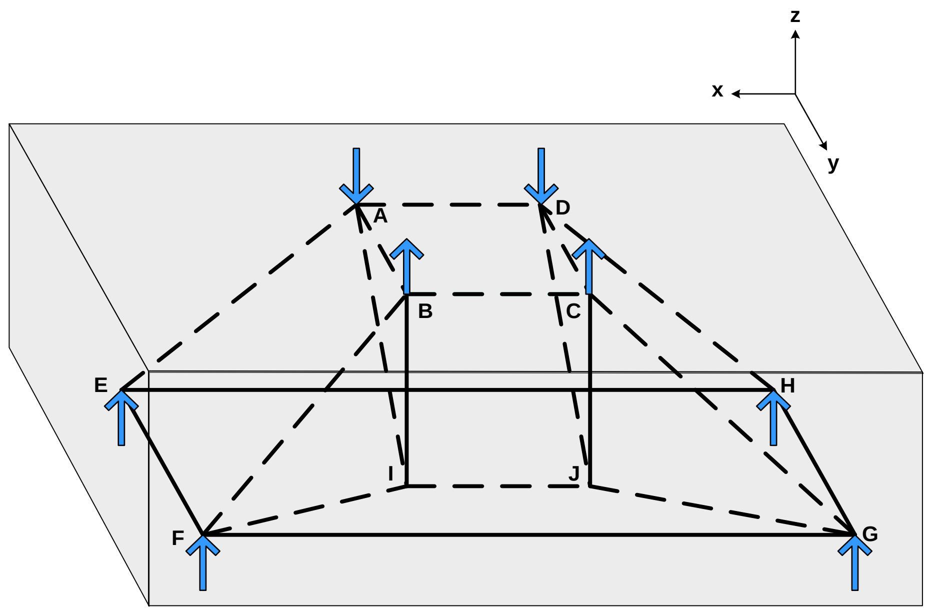


Figure 4-11: Alternative Strut-and-Tie Model for Load Case 1

Creating Three-Dimensional Strut-and-Tie Models:

Using a structural analysis software package to develop three-dimensional strut-and-tie models is recommended. Truss models may be easily defined and refined until a satisfactory truss geometry is found. Multiple strut-and-tie models may exist for a given loading and geometry. Based on the unique design requirements, the designer should seek to determine which model best represents the flow of forces within the component.

Design Step 4.3 - Proportion Ties

The forces shown in Figure 4-8 will be used to proportion the horizontal and vertical ties in the footing. The bottom mat of reinforcement will be proportioned first. To be consistent with earlier assumptions, No. 11 bars will be used in both directions in the reinforcing mat.

Ties* EF *and* GH*:

The forces in Ties *EF* and *GH* are equal because of the symmetry of the loading. The number of reinforcing bars required is determined as in the previous design examples based on *AASHTO LRFD* Equations 5.8.2.3-1 and 5.8.2.4.1-1:

$$P_u \leq \phi F_y A_{st}$$

$$P_u = 925.7\ kips$$

$$925.7\ kips \leq 0.9 \times 75.0\ ksi \times A_{st}$$

$$A_{st} \geq 13.71\ in^2$$

Using No. 11 reinforcing bars:

$$No. of\ bars = \frac{13.71\ in^2}{1.56\ \frac{in^2}{bar}} = 8.8\ bars$$

Therefore, use 9 No. 11 bars.

Ties* FG *and* EH*:

Because the loading on the column is potentially reversible, the same reinforcement will be provided for Ties *FG* and *EH*. The force in Tie *FG* is greater than in Tie *EH*. Therefore Tie *FG* is used to proportion the reinforcement. Reinforcement is designed based on *AASHTO LRFD* Equations 5.8.2.3-1 and 5.8.2.4.1-1:

$$P_u \leq \phi F_y A_{st}$$

$$P_u = 1{,}182.9\ kips$$

$$1{,}182.9\ kips \leq 0.9 \times 75.0\ ksi \times A_{st}$$

$$A_{st} \geq 17.52\ in^2$$

Using No. 11 reinforcing bars:

$$No. of\ bars = \frac{17.52\ in^2}{1.56\ \frac{in^2}{bar}} = 11.2\ bars$$

Therefore, use 12 No. 11 bars for Ties *FG* and *EH*. For consistency and symmetry, use 12 No. 11 bars for Ties *EF* and *GH* as well.

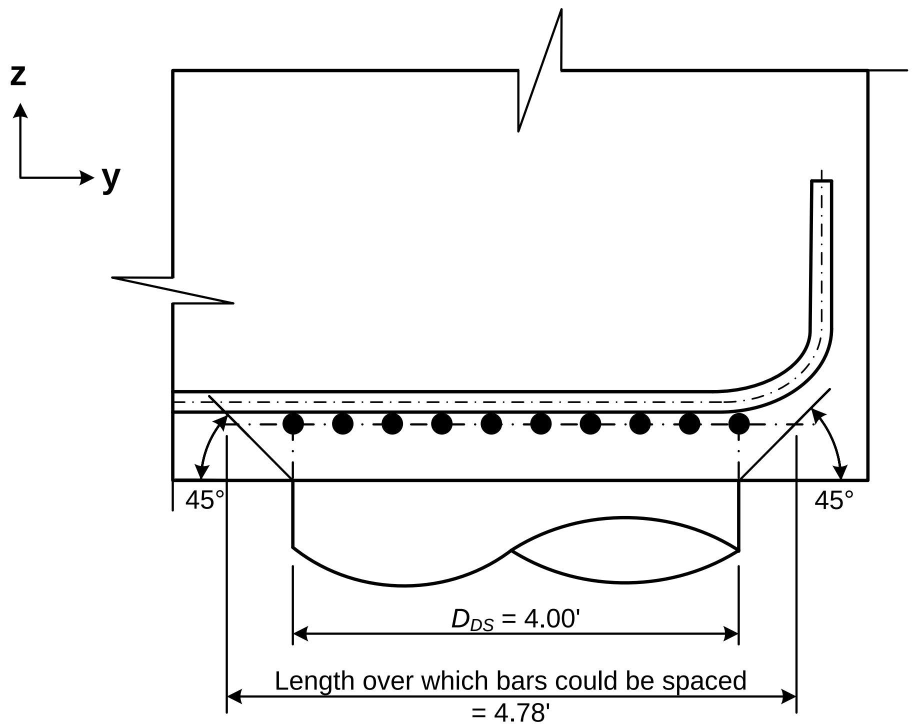


Figure 4-12: Locating Ties *FG* and *EH* Bars over Drilled Shafts

Vertical Locations of the Bottom Ties:

Referring to Figure 4-7, recall that the bottom horizontal ties were assumed to be located at the center of gravity of the bottom mat of reinforcing, where the assumption was made that the number of reinforcing bars provided in both directions was equal. As previously stated, 12 No. 11 bars will be used for Ties *FG*, *EH*, *EF*, and *GH*, for consistency and symmetry. However, if the computed value of 12 reinforcing bars was used in one direction and the computed value of 9 bars was used in the other direction, the actual location of the center of gravity of the reinforcing mat would be:

$$\frac{12\ bars \times \left(5.41\ in - \frac{1.41\ in}{2}\right) + 9\ bars \times \left(5.41\ in + \frac{1.41\ in}{2}\right)}{21\ bars}$$

$$= 5.37\ in$$

The difference between the assumed and actual center of gravity would be:

$$\Delta = 5.41\ in - 5.37\ in = 0.04\ in$$

Ties* BI *and* CJ*:

Required reinforcing in the ties is determined using *AASHTO LRFD* Equations 5.8.2.3-1 and 5.8.2.4.1-1. The forces in Ties *BI* and *CJ* are equal, so the required reinforcing is determined for each:

$$P_u \leq \phi F_y A_{st}$$

$$P_u = 339.1\ kips$$

$$339.1\ kips \leq 0.9 \times 75.0\ ksi \times A_{st}$$

$$A_{st} \geq 5.02\ in^2$$

Using No. 11 reinforcing bars:

$$No. of\ bars = \frac{5.02\ in^2}{1.56\ \frac{in^2}{bar}} = 3.2\ bars$$

Therefore, a minimum of 4 No. 11 bars should be provided. However, recall that the column face reinforcement consists of 12 No. 11 bars on each column face. This reinforcement must be continued into the footing in order to develop its capacity (either by continuing the reinforcing into the footing or using lap splices). Therefore, the 12 No. 11 bars will be able to satisfy the requirements of Ties *BI* and *CJ* (6 No. 11 bars each).

Design Step 4.4 - Perform Nodal Strength Checks

The nodal regions in three-dimensional strut-and-tie models have intricate geometries that complicate the procedure of checking them for adequate strength. Even the simplest three-dimensional nodes will have complicated geometries and would require either excessive computational time or three-dimensional CADD drafting of the node. Many attempts have been made to approximate nodal geometries in three-dimensional models (see the references at the end of this design example), but the computational effort required to accurately determine the nodal geometries severely impacts the time required to perform a strut-and-tie analysis.

Williams et al. (2012) discuss the variety of assumptions made by various researchers in the literature. These are briefly presented below. Interested readers are referred to Williams et al. (2012) and the discussed literature for additional discussion.

Typically, the nodal stresses are limited to a prescribed value. Various limits on the stresses include:

- Assume that all of the nodal regions are sufficiently strong so long as the bearing stresses at the column(s) and piles are limited to 0.85f'_c.
- Limit bearing stresses to 0.4f'_c or 0.6f'_c along with providing proper reinforcement detailing in the nodal regions.
- Limit bearing stresses to 1.0f'_c, but only if the shear span-to-depth ratio (*a/d*) is about equal to 1.0.

Other research concludes that a nodal bearing stress limit may not be a good indicator of the footing strength. Conclusions that may be drawn from a literature review are:

- Research in this area of strut-and-tie design does not recommend determining the actual nodal geometries, recognizing the fact that such a procedure would be computationally cumbersome, and
- A majority of the literature recommends adopting a design method that requires a limit on bearing stresses combined with proper reinforcement detailing.

The procedure that will be used in this design example is as follows:

- Position all nodes within the confines of the footing or pile cap. In particular, nodes directly under columns should not be positioned at the column-to-footing interface.
- Limit the concrete bearing stress on the footing or pile cap to vf'_c, where v is the concrete efficiency factor defined in *AASHTO LRFD* Article 5.8.2.5.3. Because the value of v is limited to a maximum of 0.85, this stress limit is in line with the recommendations from existing research.
- Omit the confinement modification factor, m, for additional conservatism.

The introduction of these stress limits simplifies the analysis by not requiring determination of the nodal geometries. Detailed calculations for strut-to-node interfaces are provided in the other design examples and in the accompanying training course.

Referring to Figure 4-8, the greatest bearing stresses will occur at Nodes *A, D, E,* and *H* because of the large magnitudes of the compressive forces in Struts *AE, AD,* and *DH*. These nodes will be checked using the procedure detailed above.

Check Nodes* E *and* H *(CTT):

Due to symmetry, the forces and thus the bearings areas at Nodes *E* and *H* are the same and require only one design check.

The bearing area of one of the 4.00 ft diameter drilled shafts is:

$$A_{cn} = \frac{\pi}{4} \times (D_{DS})^2 = \frac{\pi}{4} \times (48.00\ in)^2 = 1{,}809.56\ in^2$$

Because Nodes *E* and *H* are CTT nodes, the corresponding concrete efficiency factor is determined using *AASHTO LRFD* Table 5.8.2.5.3a-1:

$$0.45 \le v \le 0.65$$

$$v = 0.85 - \frac{f'_c}{20\ ksi} = 0.85 - \frac{5.0\ ksi}{20\ ksi} = 0.60$$

The bearing force to be resisted is taken as the reaction at the drilled shafts, or 1,165.0 kips. The allowable bearing force at the nodes is determined based on *AASHTO LRFD* Equation 5.8.2.5.1-1, Modified:

$$\phi P_n = \phi v f_c' A_{cn}$$

$$P_u = 1{,}165.0\ kips$$

$$f_{cu} = vf'_c = 0.60 \times 5.0\ ksi = 3.00\ ksi$$

Note the omission of the confinement modification factor, *m*, in *AASHTO LRFD* Equation 5.8.2.5.1-1.

$$\phi P_n = 0.7 \times 3.00\ ksi \times 1{,}809.56\ in^2 = 3{,}800.1\ kips$$

$$3{,}800.1\ kips > 1{,}165.0\ kips \quad \boldsymbol{OK}$$

Hence, the nodal strength is adequate according to the proposed procedure.

Nodes* A *and* D *(CCC):

Due to symmetry, the forces and bearing areas at Nodes *A* and *D* are identical. The locations of the loads are assumed to be at the centers of the shaded bearing areas shown in Figure 4-6. The bearing area for each node is:

$$A_{cn} = lw = 3.44\ ft \times \left(\frac{6.25\ ft}{2}\right) = 10.75\ ft^2 = 1{,}548.0\ in^2$$

Nodes *A* and *D* are CCC nodes, and the strengths of their bearing areas may be determined using *AASHTO LRFD* Table 5.8.2.5.3a-1 and *AASHTO LRFD* Equation 5.8.2.5.1-1, Modified:

$$\phi P_n = \phi v f_c' A_{cn}$$

$$P_u = 1{,}763.6\ kips$$

$$f_{cu} = vf'_c = 0.85 \times 5.0\ ksi = 4.25\ ksi$$

$$\phi P_n = 0.7 \times 4.25\ ksi \times 1{,}548.0\ in^2 = 4{,}605.3\ kips$$

$$4{,}605.3\ kips > 1{,}763.6\ kips \quad \boldsymbol{OK}$$

Hence, the nodal strength is adequate according to the proposed procedure.

Since these critical nodal strengths are sufficient to resist the applied loads, all nodal strengths in the strut-and-tie are adequate to resist the applied loads (Adebar, 2004, Widianto and Bayrak, 2011, and Schlaich et al., 1987).

Design Step 4.5 - Proportion Shrinkage and Temperature Reinforcement

The minimum crack control reinforcement requirements do not apply to slabs and footings per *AASHTO LRFD* Article 5.8.2.6. However, shrinkage and temperature reinforcement should still be provided to control cracking. Shrinkage and temperature reinforcement may be determined using the provisions of *AASHTO LRFD* Article 5.10.6. The required area of reinforcing on each face and in each direction is determined based on *AASHTO LRFD* Equations 5.10.6-1 and 5.10.6-2:

$$A_s \geq \frac{1.30bh}{2(b+h)f_y}$$

limited as follows:

$$0.11\,\frac{in^2}{ft} \leq A_s \leq 0.60\,\frac{in^2}{ft}$$

where:

- A_s = area of reinforcement in each direction on each face, in²/ft
- b = least width of component section, in
- h = least thickness of component section, in
- f_y = minimum yield strength of reinforcement ≤ 75.0 ksi

For the drilled shaft footing, the value of *b* is taken as 16.00 ft, or 192.00 in, and the value of *h* is taken as 5.00 ft, or 60.00 in. Therefore, the required reinforcement is determined:

$$A_s \geq \frac{1.30bh}{2(b+h)f_v} = \frac{1.30 \times 192.00\ in \times 60.00\ in}{2 \times (192.00\ in + 60.00\ in) \times 75.0\ ksi} = 0.40\,\frac{in^2}{ft}$$

Providing 1 No. 6 bar per foot yields 0.44 in²/ft, which is acceptable.

No. 6 bars will be provided on all faces except for the bottom face, where No. 11 bars will be spaced evenly between the drilled shafts. The maximum spacing of the temperature and shrinkage reinforcement is 12.0 in for footings greater than 18.0 in thick per *AASHTO LRFD* Article 5.10.6. Because the areas provided for a No. 6 bar or No. 11 bar spaced at 12.0 in exceeds the area of steel required, the maximum spacing requirement controls.

Design Step 4.6 - Provide Necessary Anchorage for Ties

Each tie in the strut-and-tie model must be fully developed at the point where the center of gravity of the reinforcement exits the extended nodal zone, in accordance with *AASHTO LRFD* Article 5.10.8.2. The three-dimensional nodes and extended nodal zones are difficult to define, making definitive calculation of the available development length impossible. Therefore, the designer must decide on an approach to finding a conservative estimate of the available development length.

Ties* EF, FG, GH*, and* EH*:

A conservative assumption will be made to estimate the available development length in relation to the dimensions of the drilled shafts. The circular drilled shafts will be idealized as square shafts of the same cross-sectional area, and the critical section for development will be taken as the interior edge of an equivalent square shaft (refer to Figure 4-13).

The side dimension, l_b, of the equivalent square shaft is given by:

$$A_{DS} = \frac{\pi}{4} \times (D_{DS})^2 = \frac{\pi}{4} \times (48.00\ in)^2 = 1{,}809.56\ in^2$$

$$l_b = \sqrt{A_{DS}} = \sqrt{(1{,}809.56\ in^2)} = 42.54\ in$$

Now that the location of the critical section has been defined, the available length is determined by:

$$l_a = OH + \frac{D_{DS}}{2} + \frac{l_b}{2} - Clear\ Cover$$

where *OH* is the distance that the footing overhangs the drilled shaft. Therefore:

$$l_a = 9.00\ in + \frac{48.00\ in}{2} + \frac{42.45\ in}{2} - 3.00\ in = 51.23\ in$$

If straight reinforcing bars are to be used, the required tension development length is computed based on *AASHTO LRFD* Equation 5.10.8.2.1a-1:

$$l_d = l_{db} \times \left(\frac{\lambda_{rl} \times \lambda_{cf} \times \lambda_{rc} \times \lambda_{er}}{\lambda}\right)$$

where l_{db} is the *basic development length*, defined by *AASHTO LRFD* Equation 5.10.8.2.1a-2:

$$l_{db} = 2.4 d_b \frac{f_y}{\sqrt{f'_c}}$$

Examining *AASHTO LRFD* Article 5.10.8.2.1b, no modification factors are required which would increase the required development length (uncoated, or black, reinforcing bars are assumed). Conservatively, the calculated development length will not be reduced as allowed by *AASHTO LRFD* Article 5.10.8.2.1c. Thus, for No. 11 reinforcing bars:

$$l_d = 2.4 d_b \frac{f_y}{\sqrt{f'_c}} \times \left(\frac{\lambda_{rl} \times \lambda_{cf} \times \lambda_{rc} \times \lambda_{er}}{\lambda} \right)$$

$$l_d = 2.4 \times 1.41\ in \times \frac{75.0\ ksi}{\sqrt{5.0\ ksi}} \times (1.0) = 113.50\ in$$

$$113.50\ in > 51.23\ in \quad \textbf{NO GOOD}$$

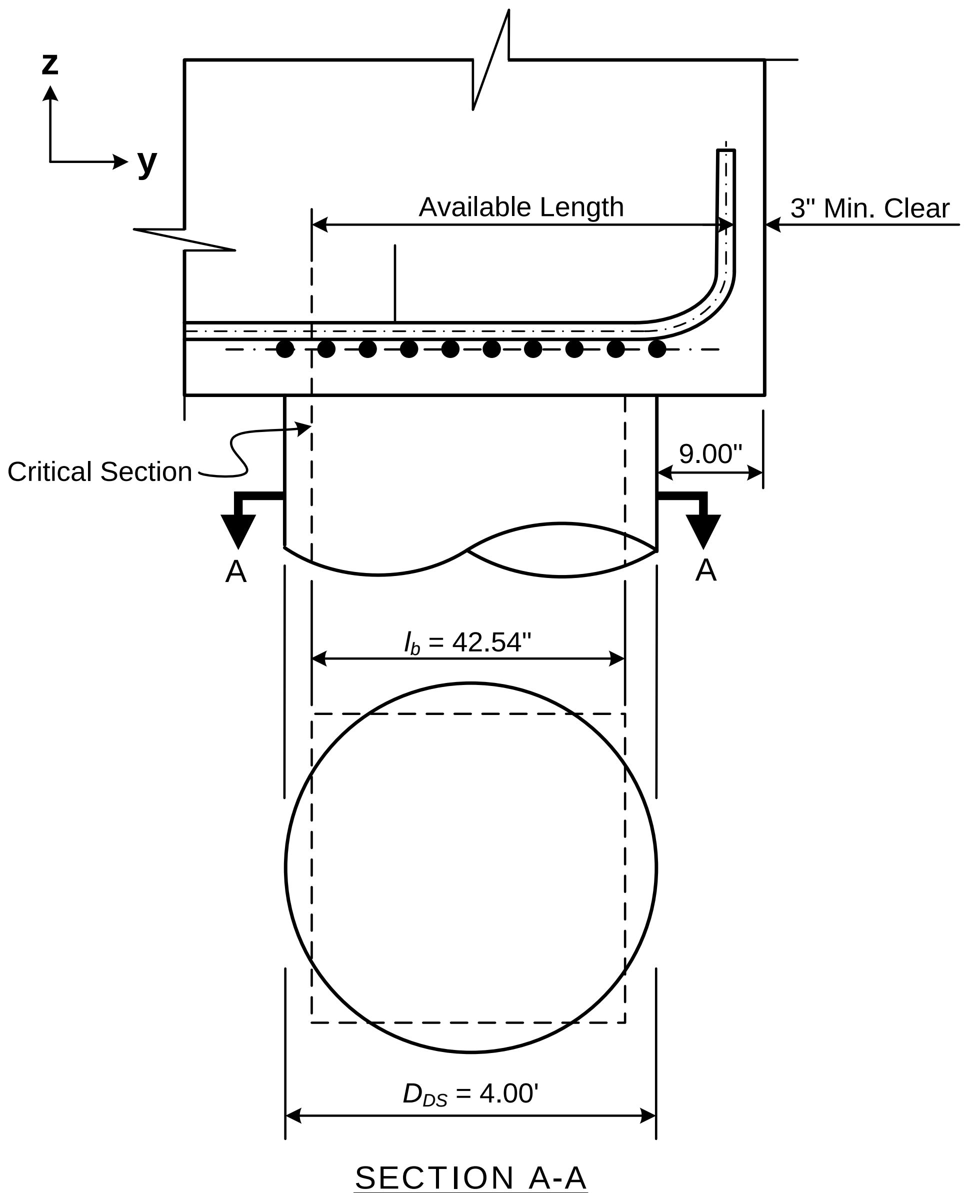


Figure 4-13: Available Development Length over the Drilled Shafts

The required development length is greater than the available length. Therefore, try providing hooked ends and checking the required development length for a hooked bar. *AASHTO LRFD* Equations 5.10.8.2.4a-1 and 5.10.8.2.4a-2 are as follows:

$$l_{dh} = l_{hb} \times \left(\frac{\lambda_{rc} \times \lambda_{cw} \times \lambda_{er}}{\lambda}\right)$$

where:

$$l_{hb} = \frac{38 d_b}{60} \times \left(\frac{f_y}{\sqrt{f_c'}}\right)$$

Based on *AASHTO LRFD* Article 5.4.2.8, the concrete density modification factor, Lambda (λ), equals 1.0 for normal weight concrete. Consequently, l_{dh} can be computed as follows:

$$l_{dh} = \frac{38 \times 1.41\ in}{60} \times \left(\frac{75.0\ ksi}{\sqrt{5.0\ ksi}}\right) \times (1.0) = 29.95\ in$$

$$29.95\ in < 51.23\ in \quad \boldsymbol{OK}$$

Adequate length is available to develop a hooked reinforcing bar in tension; therefore, provide standard hooks at the ends of the bottom mat reinforcing.

Ties* BI *and* CJ*:

Vertical Ties *BI* and *CJ* consist of the reinforcing bars extending from the column into the footing. Standard practice accomplishes this by using “L”-shaped bars extending from the footing lapped with straight bars in the column. The required development length for No. 11 reinforcing bars with a 90-degree hook was calculated as 29.95 in. The tie reinforcement must be fully developed at the point where the center of gravity of the tie reinforcement leaves the extended nodal zone.

Unfortunately, the depths of the nodal regions (and by extension, the extended nodal regions) cannot be determined with certainty because Nodes *I* and *J* are smeared nodes; they have no bearing plates or geometric boundaries which define their limits (refer to Figure 4-14). The available development length is therefore unknown. Considering that hooked reinforcing bars have been used successfully in practice for many years, it is assumed that the hooked No. 11 bars will provide adequate development for Ties *BI* and *CJ*. Because the loading is potentially reversible, all of the vertical column reinforcement entering the footing will terminate in standard 90-degree hooks.

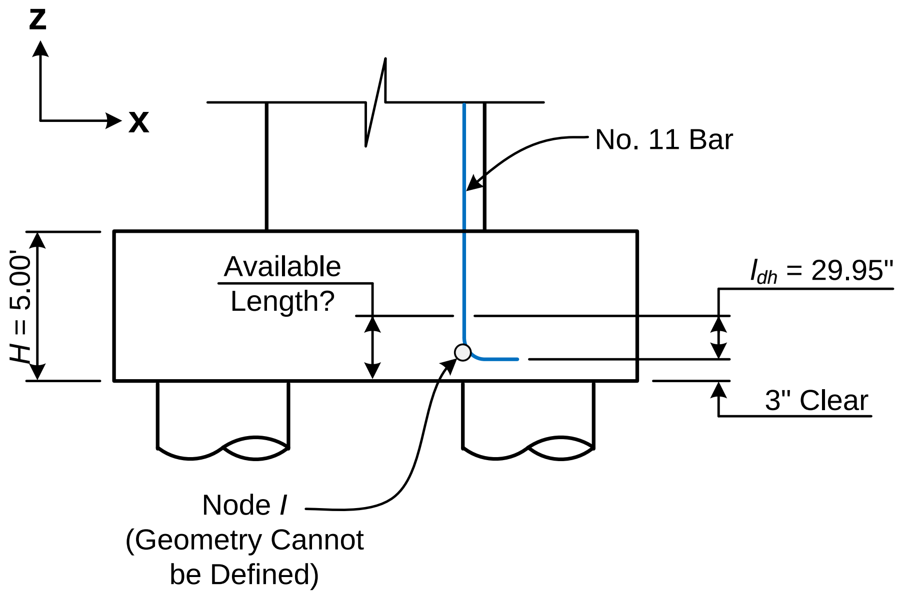


Figure 4-14: Vertical Tie Unknown Available Development Length

Design Step 5 - Analyze Structural Components (Load Case 2)

Now that the design steps have been completed for Load Case 1, the same procedure will be used for Load Case 2. Since the same procedure is used, any differences between the designs are noted.

Design Step 5.1 - Determine Loads

Refer back to Figure 4-3 for the loadings of Load Case 2. The axial force and bending moment are resolved into equivalent forces that will be applied to the strut-and-tie model, which is analogous to the method used in Design Step 4.1.

The linear stress distributions resulting from the applied loads are shown in Figure 4-15. The equivalent force system again consists of four vertical forces which correspond to the drilled shaft reactions. However, now two of the forces are compressive and two are tensile. The compressive forces act at the compressive stress resultant of the linear stress diagram. The compressive forces act a distance of 1.47 ft from the left face of the column and at the quarter-points of the column depth from the top and bottom faces. The positions of the tensile resultants are the same as in Design Step 4.1: each tensile resultant is located at the center of gravity of a group of 6 No. 11 column bars on the tension face of the column.

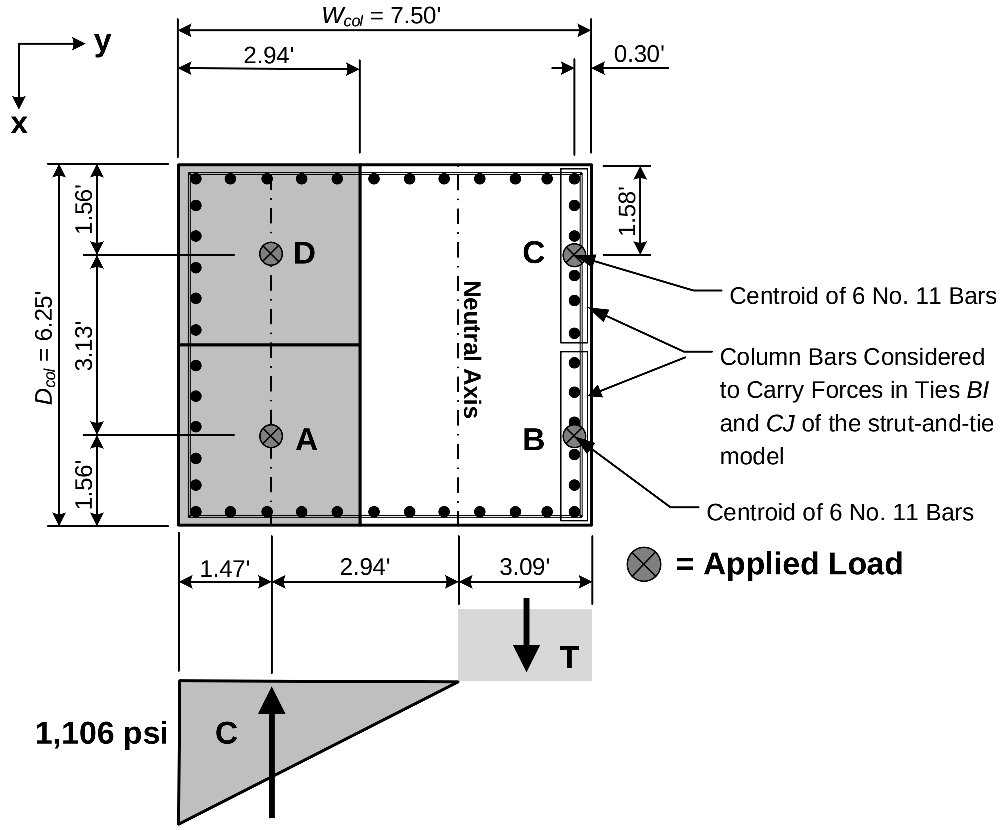


Figure 4-15: Linear Stress Distribution over Column Cross-Section and Equivalent Force System Load Locations

Equilibrium must be established once again to determine the magnitude of the compressive and tensile forces applied by the column. In the following system of equations, F_{comp} is the total compressive force acting on the system (or, the sum of the compressive forces acting at Nodes *A* and *D*) and F_{tens} is the total tensile force acting on the system (or, the sum of the tensile forces acting at Nodes *B* and *C*).

$$\begin{cases} F_{comp} - F_{tens} = 1{,}110\ kips \\ \left(\dfrac{7.50\ ft}{2} - 1.47\ ft\right)F_{comp} + \left(\dfrac{7.50\ ft}{2} - 0.30\ ft\right)F_{tens} = 7{,}942\ kip-ft \end{cases}$$

Solving yields:

$$F_{comp} = 2{,}053.5\ kips$$

$$F_{tens} = 943.5\ kips$$

Using F_{comp} and F_{tens}, the four loads that will act on the strut-and-tie model from the column are determined:

$$F_A = F_D = \frac{F_{comp}}{2} = 1{,}026.8\ kips$$

$$F_B = F_C = \frac{F_{tens}}{2} = 471.8\ kips$$

These forces are shown acting on the strut-and-tie model in Figure 4-18. The drilled shaft reactions are obtained from overall equilibrium of the drilled-shaft footing under the applied loads.

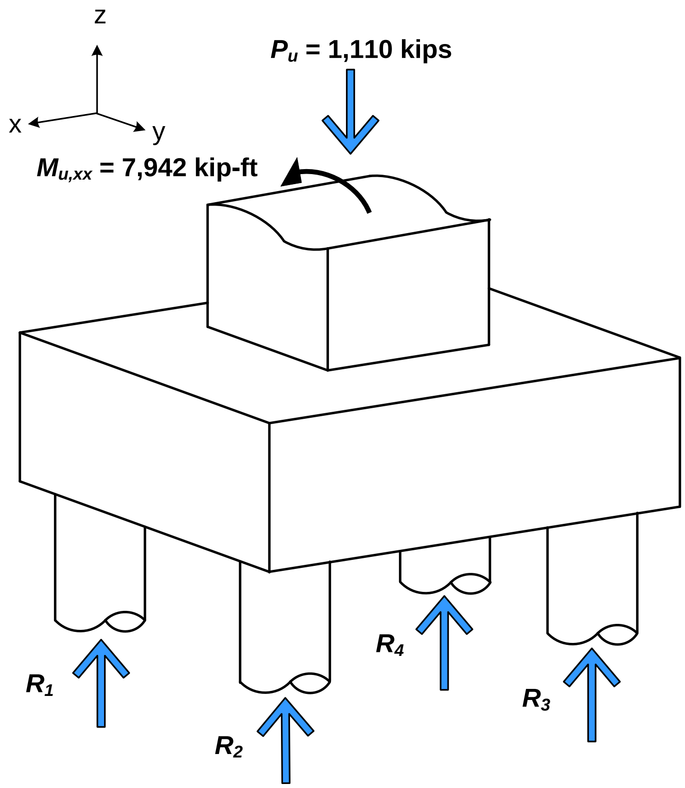


Figure 4-16: Loads Acting on Drilled Shaft Footing for Load Case 2

$$R_1 = R_4 = \frac{P_u}{4} + \frac{1}{2}\left(\frac{M_{u,xx}}{s_{Ds}}\right) = \frac{1{,}110\ kips}{4} + \frac{1}{2} \times \frac{7{,}942\ kip-ft}{10.50\ ft} = 655.7\ kips$$

$$R_2 = R_3 = \frac{P_u}{4} - \frac{1}{2}\left(\frac{M_{u,xx}}{s_{Ds}}\right) = \frac{1{,}110\ kips}{4} - \frac{1}{2} \times \frac{7{,}942\ kip-ft}{10.50\ ft} = -100.7\ kips$$

Note that the signs of R_1 and R_4 and R_2 and R_3 are opposite. By the assumed sign convention, the reactions R_2 and R_3 are tensile, indicating that the drilled shafts at Nodes *F* and *G* experience uplift under this load case.

Design Step 5.2 - Develop Strut-and-Tie Model

Development of the strut-and-tie model is based on the same methodology that was used in Design Step 4.2. The strut-and-tie model for Load Case 2 is shown in Figure 4-18, and the coordinates of each node are presented in Table 4-2.

Table 4-2: Coordinates of Nodes in Strut-and-Tie Model for Load Case 2

Node	x-Coordinate	y-Coordinate	z-Coordinate
A	9.57’	5.72’	4.60’
B	9.57’	11.45’	4.60’
C	6.44’	11.45’	4.60’
D	6.44’	5.72’	4.60’
E	13.25’	2.75’	0.45’
F	13.25’	13.25’	0.45’
G	2.75’	13.25’	0.45’
H	2.75’	2.75’	0.45’
I	9.57’	11.45’	0.45’
J	6.44’	11.45’	0.45’
K	13.25’	2.75’	4.60’
L	13.25’	13.25’	4.60’
M	2.75’	13.25’	4.60’
N	2.75’	2.75’	4.60’

Note: The origin is located in the bottom corner of the footing nearest to Node *H*.

Prior to placement of the individual struts and ties, the vertical positions of the top and bottom nodes of the strut-and-tie model must be determined. The bottom ties of the model (Ties *EF, FG, GH,* and *EH*) coincide with the center of gravity of the bottom mat of reinforcement. The distance from the bottom of the footing to these ties will be the same as for Load Case 1, or 5.41 in. In addition, a set of horizontal ties is required near the top of the footing to resist the tension created by the overturning moment. The tension reactions in two of the drilled shafts indicate the need for these ties, which will be located at the center of gravity of the top mat of reinforcement.

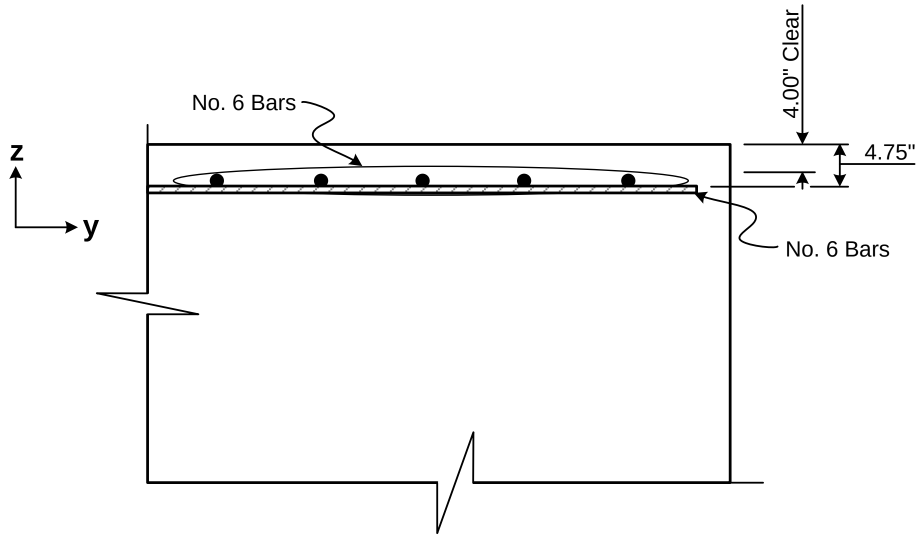


Figure 4-17: Locating Top Mat of Footing Reinforcement

The top mat of reinforcing will consist of two orthogonal layers of No. 6 reinforcing bars. An equal number of reinforcing bars will be provided in each layer and a clear cover of 4.00 in will be used, measured from the top face of the footing. Examining Figure 4-17, the center of the gravity of the top mat of reinforcing will be located at:

$$z_{top\ mat} = 4.00\ in + 0.75\ in = 4.75\ in$$

The height of the strut-and-tie model, h_{STM}, is therefore:

$$h_{STM} = 60.0\ in - 5.41\ in - 4.75\ in = 49.84\ in$$

In order to develop the rest of the strut-and-tie model, the individual struts and ties should follow the most intuitive load path, establishing equilibrium at each node. The tensile forces at Nodes *B* and *C* require vertical Ties *BI* and *CJ* to transfer the tension through the footing depth. Similarly, two additional ties (Ties *FL* and *GM*) are required to resist the tensile drilled shaft reactions at Nodes *F* and *G*. These ties are required to "anchor" the footing to the drilled shafts.

Note that Ties *BI* and *FL* together form a non-contact lap splice, which would tie the reaction at Node *F* to the applied load at Node *B*. Thus compressive stress will develop between these two nodes, requiring a strut to transfer the stress between the nodes. This is idealized by Strut *IL*. The forces in Ties *CJ* and *GM* similarly require Strut *JM*.

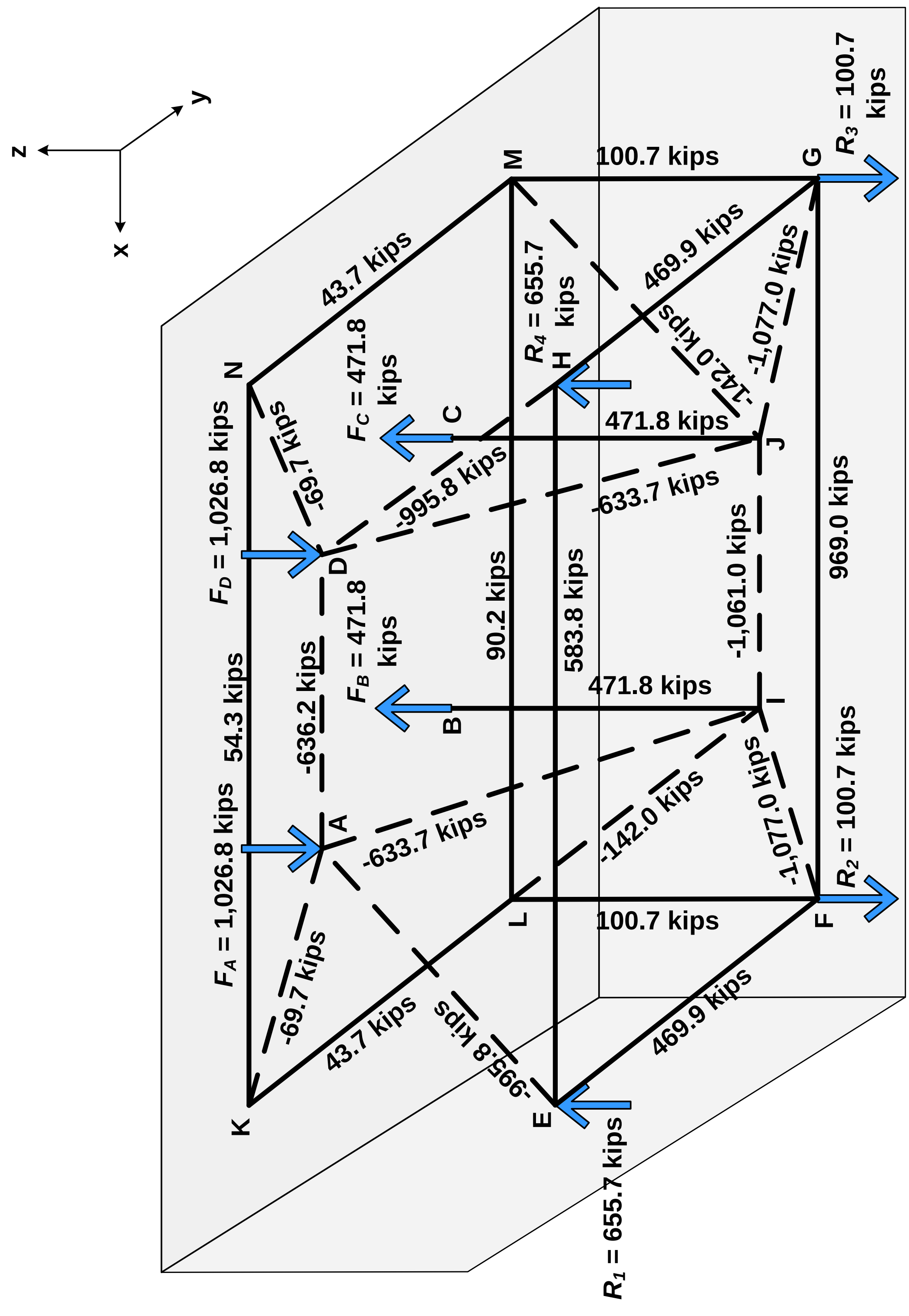


Figure 4-18: Isometric View for Strut-and-Tie Model for Load Case 2

Vertical equilibrium will be satisfied at Nodes *I* and *J* by drawings Struts *AI* and *DJ*, and Struts *AE* and *DH* satisfy equilibrium at Nodes *A* and *D*. Because of the compressive stresses from these struts flowing to the drilled shafts, tension develops in the bottom of the footing, which will be carried by Ties *EF, FG, GH,* and *EH*. Likewise, Struts *IL* and *JM* connecting the vertical ties create tension in the top of the footing, requiring Ties *KL, LM, MN,* and *KN*. Once again, the strut-and-tie model is checked to ensure that the angles between struts and ties are always greater than or equal to 25 degrees.

The strut-and-tie model is analyzed in the same manner as was done for Load Case 1. Recall that a linear elastic analysis of the truss model should yield the same reactions at the drilled shafts as those found in Design Step 5.1.

Vertical Locations of the Bottom Ties:

The reader is encouraged to examine Figure 4-8 and Figure 4-18 together. Note the differences and similarities in the strut-and-tie models for each load case. These models are the result of several iterations to the three-dimensional truss geometries that ultimately result in models that reflect the flow of forces in the footing. Taking time to visualize and sketch the possible flow of forces in the footing from a given loading may help reduce the time needed to modify the truss geometry.

Analysis of this strut-and-tie model is now complete.

Design Step 5.3 - Proportion Ties

The calculated forces in the strut-and-tie models of Load Case 1 and Load Case 2 should be compared to determine the controlling design forces for the struts and ties. The bottom tie forces (Ties *EF, FG, GH,* and *EH*) in Load Case 1 control, so design of those ties will not be reexamined. The vertical tie forces (Ties *BI* and *CJ*) for Load Case 2 control, so these ties will be redesigned. The remaining ties (Ties *FL, GM, KL, LM, MN,* and *KN*) are unique to Load Case 2 and must be designed.

Ties* KL, LM, MN*, and* KN*:

The force in Tie *LM* controls the design of the top horizontal ties. The amount of reinforcing required is found using *AASHTO LRFD* Equations 5.8.2.3-1 and 5.8.2.4.1-1:

$$P_u \leq \phi F_y A_{st}$$

$$P_u = 90.2\ kips$$

$$90.2\ kips \leq 0.9 \times 75.0\ ksi \times A_{st}$$

$$A_{st} \geq 1.34\ in^2$$

Using No. 6 reinforcing bars:

$$No.of\ bars = \frac{1.34\ in^2}{0.44\ \frac{in^2}{bar}} = 3.0\ bars$$

Therefore, a minimum of 3 No. 6 bars are required to carry the tie force. Recall that the temperature and shrinkage steel defined in Design Step 4.5 was No. 6 bars in each direction, at about 12.0 in spacing. At a maximum spacing of 12.0 in, about four No. 6 bars are located above each drilled shaft. The number of bars available to carry the tie force is thus greater than the number required; thus, the temperature and shrinkage steel is adequate to resist the tie forces. Using even spaces, the actual spacing of the No. 6 bars is approximately 11.0 in.

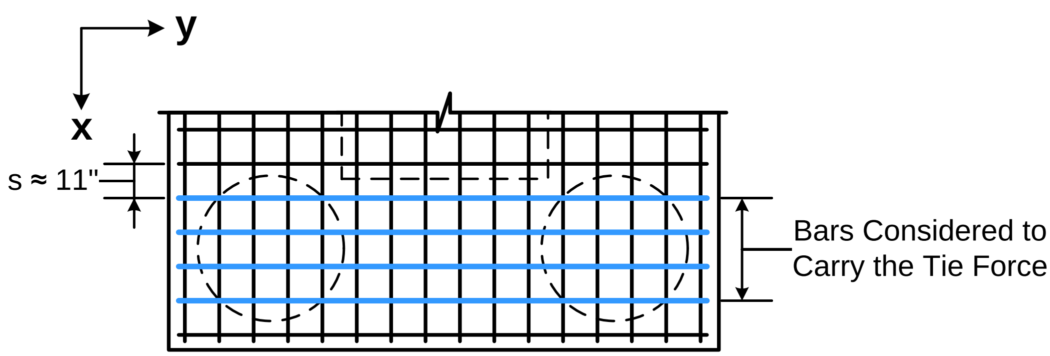


Figure 4-19: Reinforcing Carrying Forces in Ties *KL, LM, MN,* and *KN*

Ties* BI *and* CJ*:

The forces in these ties are larger for Load Case 2 than for Load Case 1. Therefore their design is re-evaluated. Considering the reinforcement from Load Case 1, 6 No. 11 bars are provided which extend from the column into the footing for each tie. The tie strength must be checked against the new demand using *AASHTO LRFD* Equations 5.8.2.3-1 and 5.8.2.4.1-1:

$$P_u = 471.8\ kips$$

$$471.8\ kips \leq 0.9 \times 75.0\ ksi \times A_{st}$$

$$A_{st} \geq 7.00\ in^2$$

Using 6 No. 11 reinforcing bars:

$$A_{st,provided} = 6\ bars \times 1.56\ \frac{in^2}{bar} = 9.36\ in^2$$

$$9.36\ in^2 > 7.00\ in^2 \quad \boldsymbol{OK}$$

Therefore, the 12 No. 11 reinforcing bars in the column (6 for each tie) are adequate to resist the tie forces.

Ties* FL *and* GM*:

Finally, the reinforcement for Ties *FL* and *GM* is defined. These ties represent the reinforcing bars which anchor the drilled shafts into the footing. The assumed layout of the reinforcement is typical of standard drilled shaft construction and is shown in Figure 4-20. No. 9 reinforcing bars are a common size reinforcing bar specified in drilled shaft design. The reinforcement in the drilled shafts at Nodes *F* and *G* will be extended into the footing in order to satisfy the reinforcement requirements for Ties *FL* and *GM*, based on *AASHTO LRFD* Equations 5.8.2.3-1 and 5.8.2.4.1-1:

$$P_u = 100.7\ kips$$

$$100.7\ kips \leq 0.9 \times 75.0\ ksi \times A_{st}$$

$$A_{st} \geq 1.49\ in^2$$

Using No. 9 reinforcing bars:

$$No. of\ bars = \frac{1.49\ in^2}{1.00\ \frac{in^2}{bar}} = 1.5\ bars$$

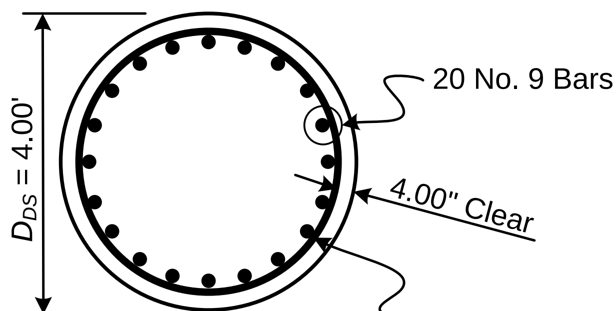


Figure 4-20: Assumed Drilled Shaft Reinforcing Layout

A minimum of 2 No. 9 drilled shaft bars must be extended into the footing. However, all of the drilled shaft reinforcement will be extended into the footing, consistent with typical construction practice. However, this reinforcement must be adequately anchored in order to contribute to the strength of Ties *FL* and *GM*. This requirement will be checked in Design Step 5.6.

Design Step 5.4 - Perform Nodal Strength Checks

Recall the discussion in Design Step 4.4 regarding the complicated geometries of nodal regions in three-dimensional strut-and-tie models. The simplified nodal strength procedure that was posited in that discussion will be applied in this design step.

Comparing the truss member forces in the strut-and-tie models for Load Case 1 and Load Case 2, the compressive forces bearing on the footing are greater for Load Case 1 than for Load Case 2. Therefore, the compressive bearing stress checks for Load Case 2 will not control design of the nodal regions, so no further strength checks are required.

Design Step 5.5 - Proportion Shrinkage and Temperature Reinforcement

The required temperature and shrinkage reinforcement was determined in Design Step 4.5 and does not need to be revisited.

Design Step 5.6 - Provide Necessary Anchorage for Ties

Proper anchorage of the bottom mat reinforcement (Ties *EF, FG, GH,* and *EH*) and vertical Ties *BI* and *CJ* was discussed in Design Step 4.6. These ties will be sufficiently anchored with the use of 90-degree hooks. Anchorage of the ties unique to Load Case 2 (Ties *KL, LM, MN, KN, FL,* and *GM*) is discussed in this design step.

Ties* KL, LM, MN, *and* KN*:

The horizontal top mat reinforcement must be properly anchored at Nodes *K, L, M,* and *N*. These four nodes are smeared nodes with no bearing plates or boundaries that define their geometries. Thinking three-dimensionally, the diagonal struts that enter these four nodes (Struts *AK, DN, IL,* and *JM*) will create large extended nodal zones; so, to maintain conservatism, the critical section for development of the reinforcement is assumed to be the same as the bottom mat of reinforcement. This location is the plane above the edge of the equivalent square drilled shaft that was determined in Design Step 4.6. The available development length is therefore the same as for the bottom reinforcement, or 51.23 in. Note that this requires 3.00 in of clear cover at each end of the reinforcing bars.

The required development length of a straight No. 6 bar is now checked. Checking *AASHTO LRFD* Articles 5.10.8.2.1b and 5.10.8.2.1c, there are modification factors applicable to this reinforcement. These are lambda sub r l (λ_{rl}) and lambda sub r c (λ_{rc}).

λ_{rl} = horizontal reinforcement placed such that more than 12.0 in of concrete is cast below it, $\lambda_{rl} = 1.3$

λ_{rc} = reinforcement confinement factor, determined according to the provisions of *AASHTO LRFD* Article 5.10.8.2.1c below:

The reinforcement confinement factor, is limited as follows, based on *AASHTO LRFD* Equations 5.10.8.2.1c-1, 5.10.8.2.1c-2, and 5.10.8.2.1c-3:

$$0.4 \leq \lambda_{rc} \leq 1.0$$

where:

$$\lambda_{rc} = \frac{d_b}{c_b + k_{tr}}$$

$$k_{tr} = \frac{40A_{tr}}{sn}$$

where:

- c_b = the smaller of the distance from the center of the bar being developed to nearest concrete surface and one-half of the center-to-center spacing of the bars being developed, in
- k_{tr} = transverse reinforcement index
- A_{tr} = total cross-sectional area of transverse reinforcement in spacing s which crosses the potential plane of splitting of the reinforcing being developed, in^2
- s = maximum center-to-center spacing of transverse reinforcement within l_d, in
- n = number of bars developed along the plane of splitting

Illustrations of these variables are given in *AASHTO LRFD* Figure C5.10.8.2.1c-1. The value of c_b is found thus:

$$c_b = MIN\begin{cases} 4.00\ in\ clear + \dfrac{0.75\ in}{2} = 4.38\ in \\ \dfrac{1}{2} \times bar\ spacing = \dfrac{1}{2} \times 12.0\ in = 6.0\ in \end{cases} = 4.38\ in$$

The spacing s of the transverse reinforcement is equal to 12.0 in (recall there are No. 6 bars spaced at about 12.0 in in both directions), therefore the area A_{tr} = 0.44 in^2. The number of bars developed along the plane of splitting is taken as 1. Therefore:

$$k_{tr} = \frac{40A_{tr}}{sn} = \frac{40 \times 0.44\ in^2}{12.0\ in \times 1} = 1.47\ in$$

$$\lambda_{rc} = \frac{d_b}{c_b + k_{tr}} = \frac{0.75\ in}{4.38\ in + 1.47\ in} = 0.13\ \ \therefore \lambda_{rc} = 0.4$$

The other development length modification factors are taken as 1.0. Therefore, the required development length of the No. 6 bars is computed as follows, based on *AASHTO LRFD* Equations 5.10.8.2.1a-1 and 5.10.8.2.1a-2:

$$l_d = 2.4 d_b \frac{f_y}{\sqrt{f'_c}} \times \left(\frac{\lambda_{rl} \times \lambda_{cf} \times \lambda_{rc} \times \lambda_{er}}{\lambda} \right)$$

$$l_d = 2.4 \times 0.75\ in \times \frac{75.0\ ksi}{\sqrt{5.0\ ksi}} \times \left(\frac{1.3 \times 1.0 \times 0.4 \times 1.0}{1.0} \right) = 31.39\ in$$

$$31.39\ in < 51.23\ in \quad \boldsymbol{OK}$$

Therefore, proper anchorage is provided for the straight No. 6 bars with 3.00 in of clear cover at each end.

Ties* FL *and* GM*:

Ties *Fl* and *GM* must be properly anchored at Nodes *L* and *M*. In Design Step 5.3, it was determined that a minimum of 2 No. 9 bars must extend from the drilled shafts into the footing to satisfy the reinforcement requirements for these ties. Considering typical drilled shaft construction, all of the drilled shaft reinforcing bars will extend into the footing. The required development length for a No. 9 reinforcing bar in tension is computed based on *AASHTO LRFD* Equations 5.10.8.2.1a-1 and 5.10.8.2.1a-2:

$$l_d = 2.4 d_b \frac{f_y}{\sqrt{f'_c}} \times \left(\frac{\lambda_{rl} \times \lambda_{cf} \times \lambda_{rc} \times \lambda_{er}}{\lambda} \right)$$

The reinforcement location factor, λ_{rl}, coating factor, λ_{cf}, and density modification factor, λ, will be taken as 1.0. Conservatively, the reinforcement confinement factor, λ_{rc}, will also be taken as 1.0. The development length required will be reduced by the excess reinforcement modification factor, λ_{er}, determined using *AASHTO LRFD* Equation 5.10.8.2.1c-4:

$$\lambda_{er} = \frac{A_{s,required}}{A_{s,provided}}$$

The required tension reinforcement was determined in Design Step 5.3. Twenty No. 9 reinforcing bars are provided. Therefore:

$$\lambda_{er} = \frac{A_{s,required}}{A_{s,provided}} = \frac{1.49\ in^2}{20.00\ in^2} = 0.08$$

Hence:

$$l_d = 2.4 \times 1.13\ in \times \frac{75.0\ ksi}{\sqrt{5.0\ ksi}} \times \left(\frac{1.0 \times 1.0 \times 1.0 \times 0.08}{1.0} \right) = 7.26\ in$$

Similar to Nodes *I* and *J*, Nodes *L* and *M* are smeared nodes whose geometries cannot be determined. Visualizing the three-dimensional geometries of the nodal regions, the straight No. 9 bars should be adequate to anchor the ties if the ends of the bars are extended as close as practical to the top of the footing. The ends of the bars should maintain the 4.00 inch minimum clear cover at the top face of the footing.

The No. 9 bars will be extended in all four drilled shafts considering constructability concerns and the potential reversibility of the applied loads. The geometry of the drilled shaft reinforcing is shown in Figure 4-21.

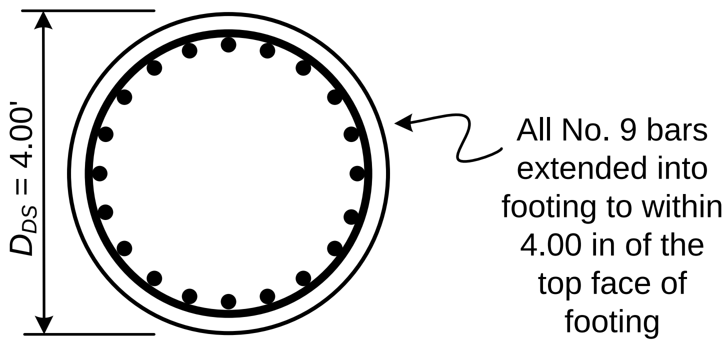


Figure 4-21: Longitudinal Drilled Shaft Reinforcing

Design Step 6 - Draw Reinforcement Layout

At this point, the strut-and-tie analysis of the drilled shaft footing is complete. Sketches of the final reinforcement layouts are provided in Figure 4-22 through Figure 4-28.

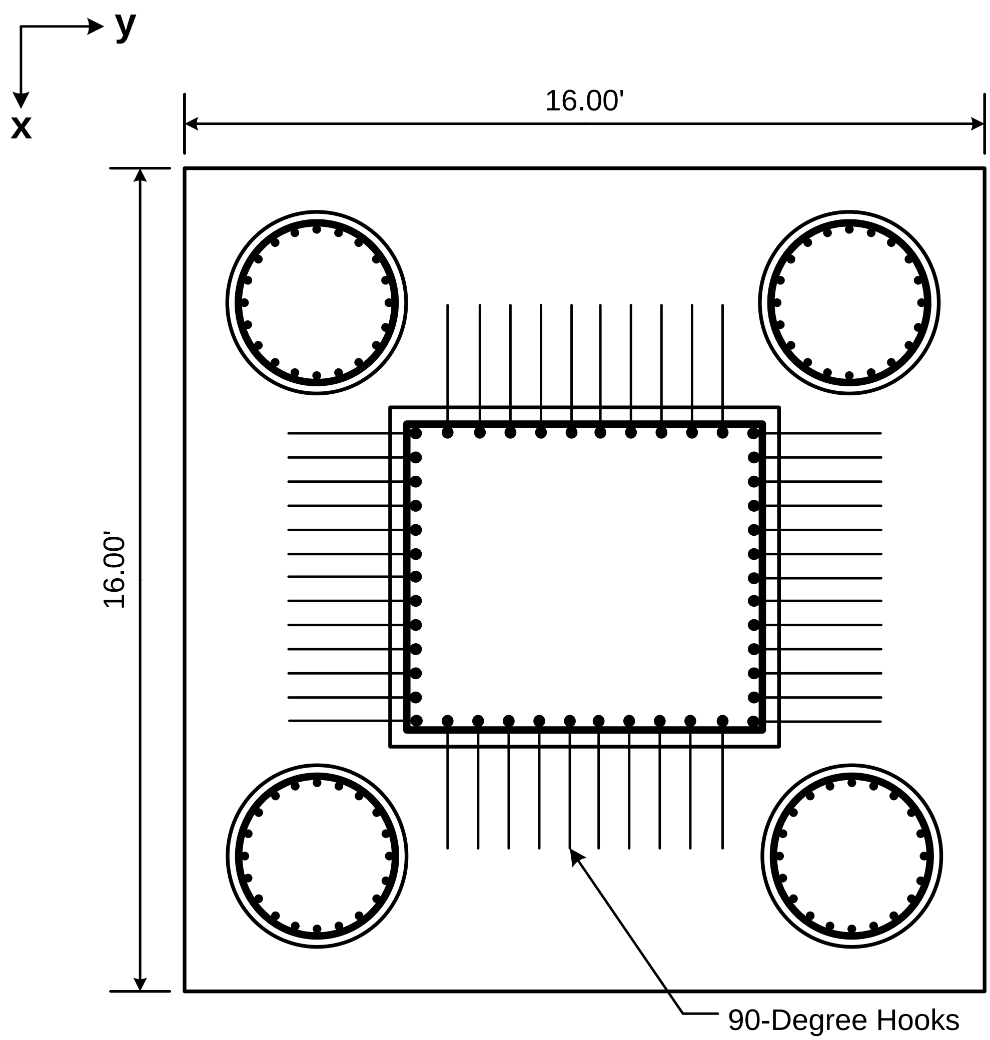


Figure 4-22: Reinforcement Details 1 - Anchorage of Ties

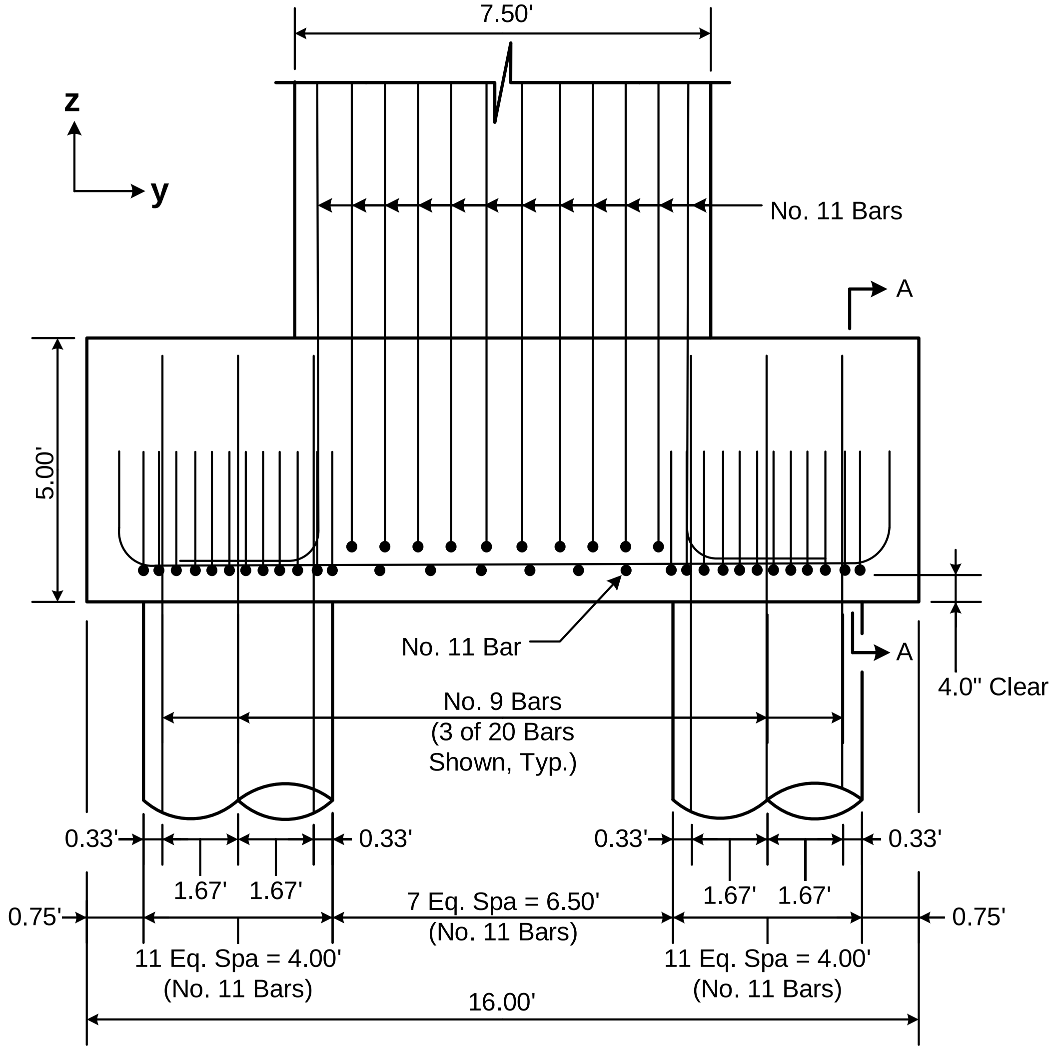


Figure 4-23: Reinforcement Details 2 - Elevation View of Primary Reinforcement

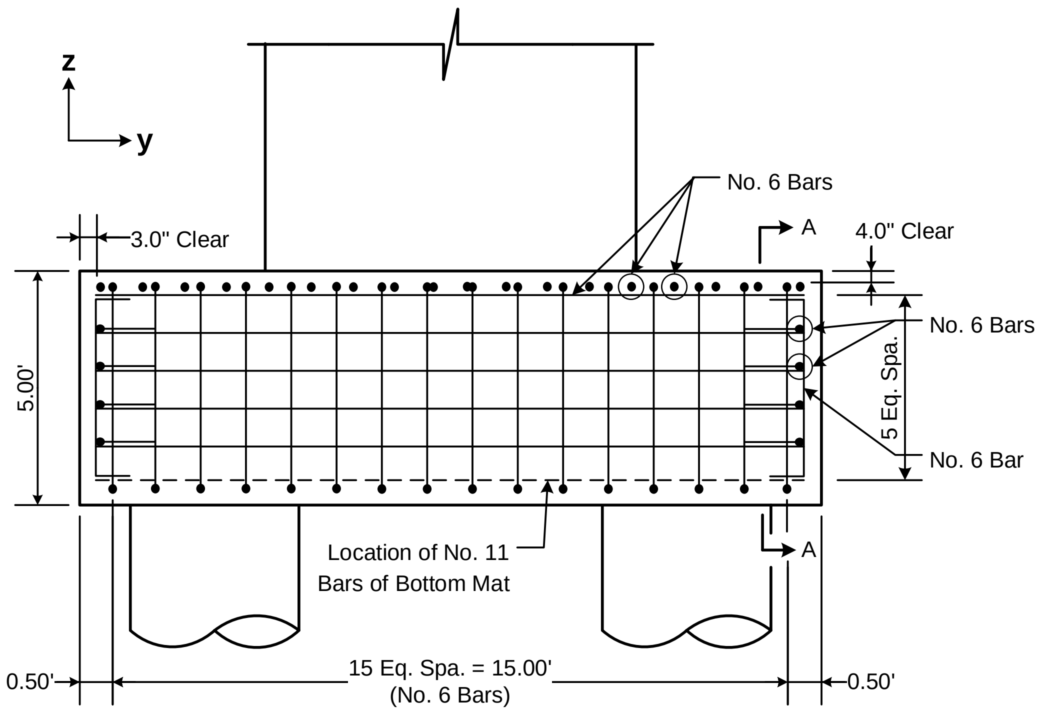


Figure 4-24: Reinforcement Details 3 - Temperature and Shrinkage Reinforcement

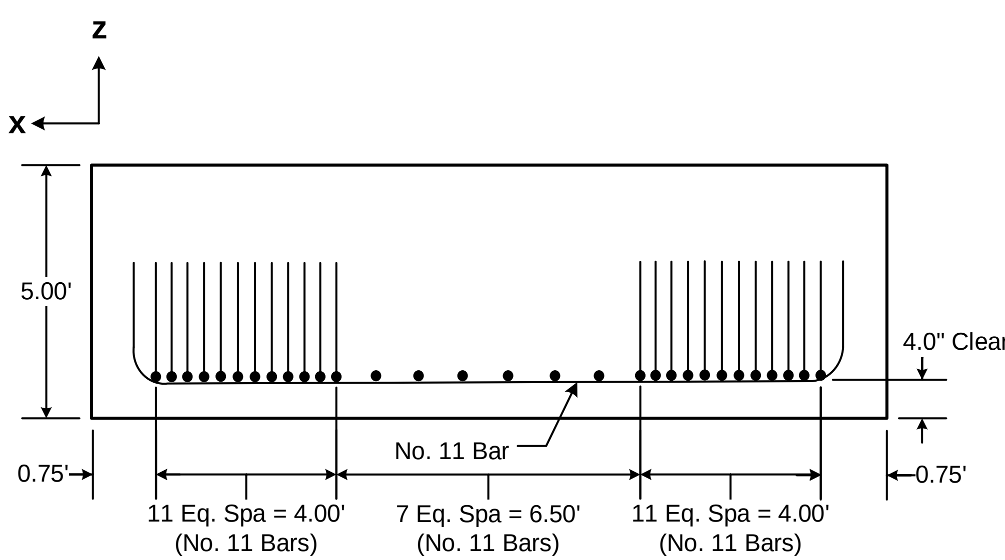


Figure 4-25: Reinforcement Details 4 - Section A-A of Figure 4-24

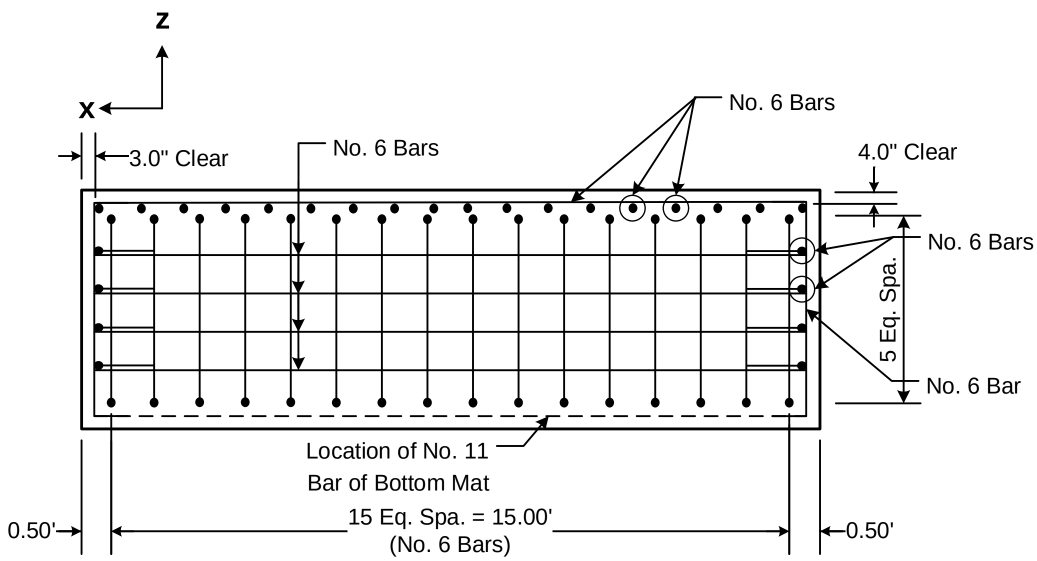


Figure 4-26: Reinforcement Details 5 - Temperature and Shrinkage Reinforcement of Section A-A in Figure 4-24

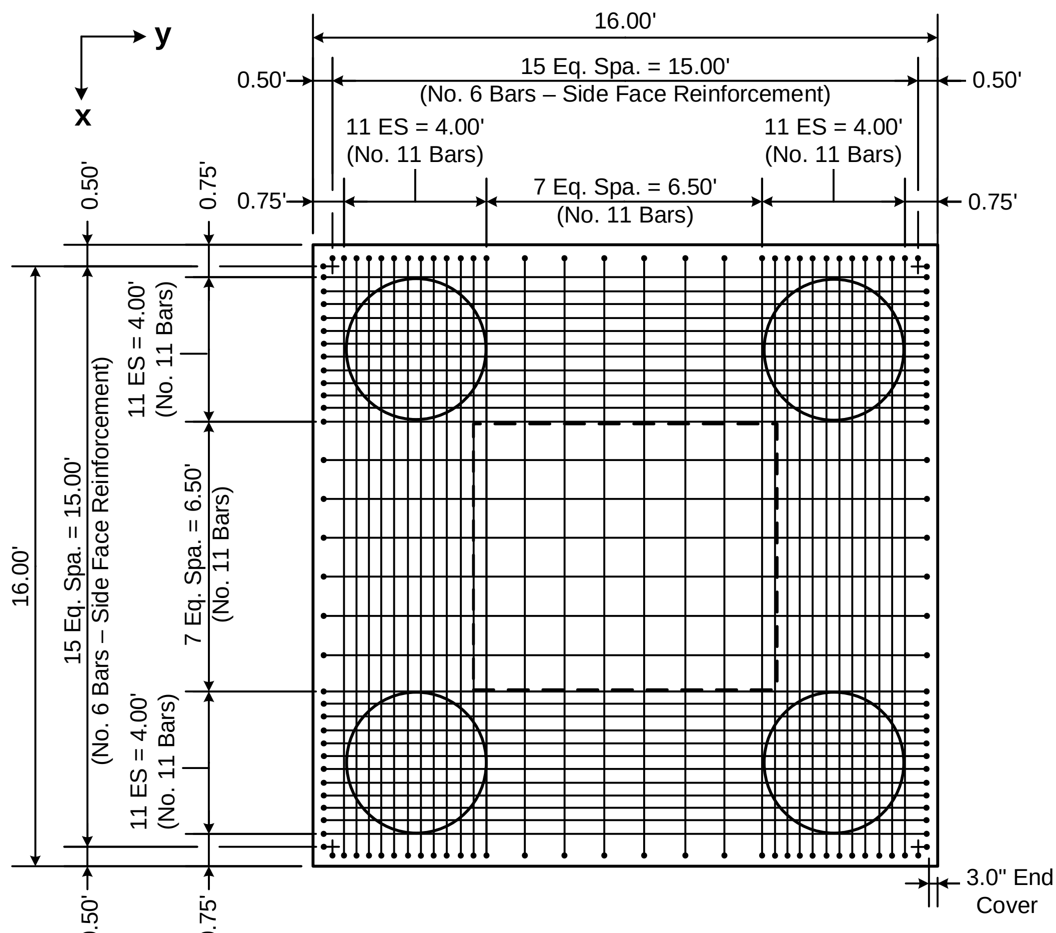


Figure 4-27: Reinforcement Details 6 - Bottom Mat Reinforcement

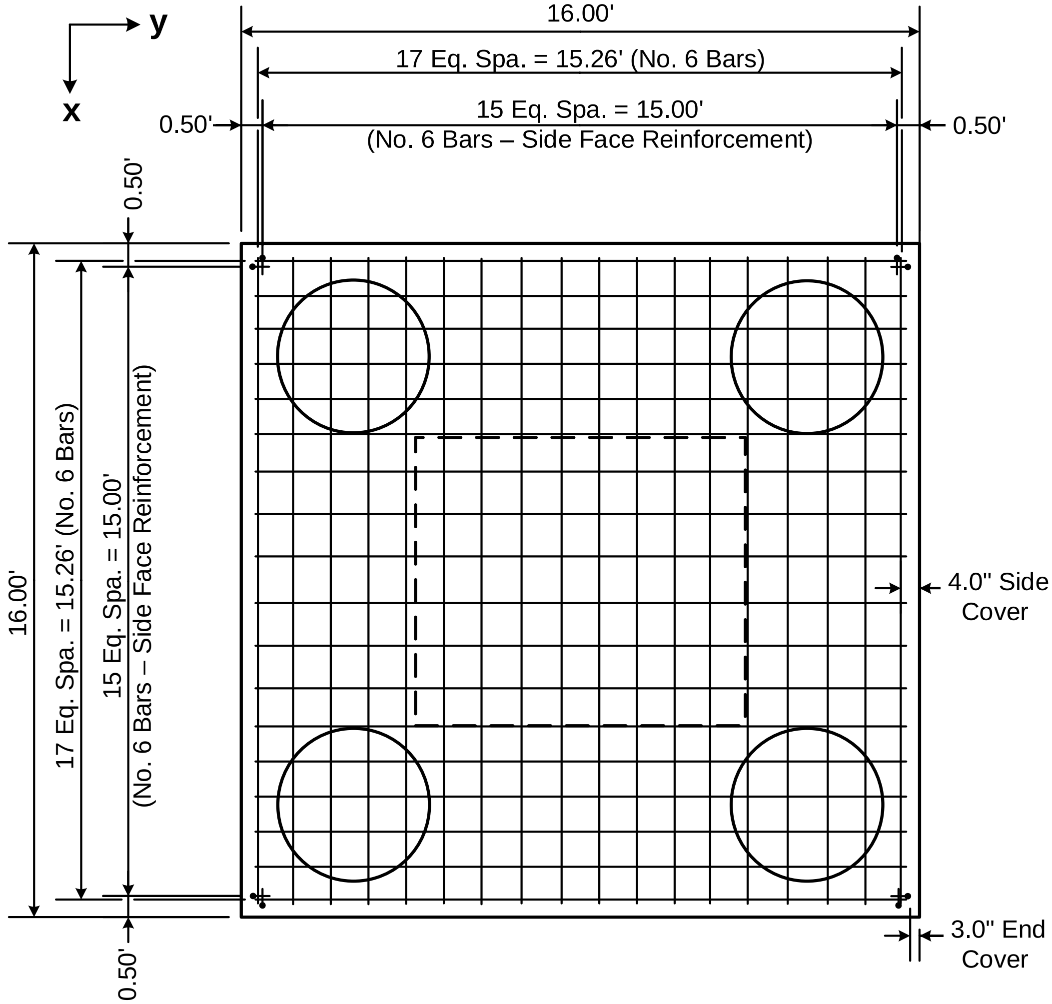


Figure 4-28: Reinforcement Details 7 - Top Mat Reinforcement

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